The Conversion of the Supply and Use Tables to Symmetric Input-Output Tables: A Critical Review[*]

George Soklis[**]

ABSTRACT

This paper reviews the available methods used to convert Supply and Use Tables of actual economic systems to Symmetric Input-Output Tables. It is argued that all conversion methods rest on the unrealistic assumption that single production, and not joint production, characterizes the economic structure of the real world. Finally, a straightforward treatment, based on general joint-product models inspired by von Neumann (1945) and Sraffa (1960), of the Supply and Use Tables is proposed as a way out of the inconsistencies of the conversion methods.

INTRODUCTION

In 1968 System of National Accounts, United Nations introduced the Supply and Use Tables (SUT hereafter) in the compilation of Input-Output Tables (United Nations, 1968). In those Tables there are industries that produce more than one commodity and commodities that are produced by more than one industry. Since then, there has been an ongoing discussion on how these tables of joint production can be converted to Symmetric Input-Output Tables (SIOT hereafter) of single production.[1]

Most of the discussion has been focused on two alternative assumptions for dealing with the problem at hand: (i) the Commodity Technology Assumption (CTA hereafter); and (ii) the Industry Technology Assumption (ITA hereafter). It can be said that the CTA is doubtless the preferred method, since it is the only one that fulfils

[*] This paper was published in *Bulletin of Political Economy, Vol. 3, No. 1, 2009: pp. 51*-70. Earlier versions of this paper were presented at a Workshop of the 'Study Group on Sraffian Economics' at the Panteion University, in September 2007, and at the '10th Conference of the Greek Historians of Economic Though' at the University of Thessalia, in May 2008. I am very grateful to Alexis Ioannides, Eleftheria Rodousaki, Nikolaos Rodousakis and, in particular, Theodore Mariolis and Lefteris Tsoulfidis for helpful comments and discussions. I am also indebted to two anonymous referees for their helpful comments on earlier versions of this paper. It goes without saying that the responsibility for the views expressed and any errors rests entirely with the author.

[**] Department of Public Administration, Panteion University, 136, Syngrou Ave, Athens 17671, Greece; Email: gsok@hotmail.gr

basic axioms of the Input-Output analysis (see Jansen and ten Raa, 1990). However, even in this method certain problems arise that cast doubt to its reliability.

The remainder of the paper is organized as follows: Section 2 reviews the alternative methods of conversion of SUT into SIOT. Section 3 offers an evaluation of these methods. Section 4 presents the von Neumann-Sraffa treatment of joint production as the way out of the problems encountered in the conversion methods. Section 5 concludes the paper.

THE CONVERSION METHODS

Let $\mathbf{U} \equiv [u_{ij}]$ be the Use matrix, which describes commodities (rows) consumed by industries (columns), $\mathbf{V} \equiv [v_{ij}]$ be the Make matrix which describes commodities (rows) produced by industries (columns),[2] $\mathbf{f}$ be the column vector of final demand, $\mathbf{v}^{\mathrm{T}}$ be the row vector of value-added ('T' is the sign for transpose) and $\mathbf{e}$ be the column summation vector (all magnitudes are expressed in money terms). Then, an *actual* joint-product system (SUT) is described by the following relations[3]

$$\mathbf{Ve} \equiv \mathbf{Ue} + \mathbf{f} \tag{1}$$

$$\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{U} + \mathbf{v}^{\mathrm{T}} \tag{2}$$

The conversion methods try to transform (1) and (2) into the following single-product system (SIOT)

$$\mathbf{x} \equiv \mathbf{Ze} + \mathbf{f}^{*} \tag{3}$$

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{Z} + (\mathbf{v}^{*})^{\mathrm{T}} \tag{4}$$

where $\mathbf{x}$ is a column vector that describes the output of commodities of the single-product system, $\mathbf{Z}(\equiv \mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}})$ is the transactions matrix, $\mathbf{A}(\mathbf{U},\mathbf{V})$ is the commodity-by-commodity direct requirements matrix that is derived from the conversion of the SUT to SIOT,[4] $\mathbf{f}^{*}$, $(\mathbf{v}^{*})^{\mathrm{T}}$ are the transformed vectors of final demand and value added, respectively, and ['$\wedge$'] denotes diagonalization either by suppression of the off-diagonal entries of a square matrix or by placement of the entries of a vector.

The usual methods of conversion of SUT to SIOT are the following:

The Commodity Technology Assumption

The CTA assumes that each industry produces only the total output of the commodity that is primary to that industry and that each commodity has its own input structure, irrespective of the industry that produces it. Thus, it holds[5]

$$\mathbf{U} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V} \qquad (5)$$

and

$$\mathbf{x} \equiv \mathbf{V}\mathbf{e} \qquad (6)$$

Consequently, the direct requirements matrix is obtained as follows

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv \mathbf{U}\mathbf{V}^{-1} \qquad (7)$$

Substituting (5) and (6) in (1) yields

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x} + \mathbf{f} \qquad (8)$$

From relations (8) and (3) it follows

$$\mathbf{f}^{*} \equiv \mathbf{f} \qquad (9)$$

Substituting (5) in (2) yields

$$\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V} + \mathbf{v}^{\mathrm{T}} \qquad (10)$$

Post-multiplying (10) by $\mathbf{V}^{-1}(\widehat{\mathbf{V}\mathbf{e}})$ gives

$$\mathbf{e}^{\mathrm{T}}(\widehat{\mathbf{V}\mathbf{e}}) \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}\mathbf{e}}) + \mathbf{v}^{\mathrm{T}}\mathbf{V}^{-1}(\widehat{\mathbf{V}\mathbf{e}}) \qquad (11)$$

Substituting (6) in (11) we obtain

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}} + \mathbf{v}^{\mathrm{T}}\mathbf{V}^{-1}(\hat{\mathbf{x}}) \qquad (12)$$

From relations (12), (4) and (6) it follows

$$(\mathbf{v}^{*})^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}}\mathbf{V}^{-1}(\widehat{\mathbf{V}\mathbf{e}}) \qquad (13)$$

Hence, under the CTA, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $\mathbf{Ve} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{Ve} + \mathbf{f}$ and $(\mathbf{Ve})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}}) + \mathbf{v}^{\mathrm{T}}\mathbf{V}^{-1}(\widehat{\mathbf{Ve}})$, respectively.

The Industry Technology Assumption

The ITA assumes that each industry produces only the total output of the commodity that is primary to that industry and that has the same input requirements for any unit of output. In that case, the input structure of each commodity depends on what industry produces it. In mathematical terms, it holds[6]

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv \mathbf{U}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}}(\widehat{\mathbf{Ve}})^{-1} \tag{14}$$

and

$$\mathbf{x} \equiv \mathbf{Ve} \tag{15}$$

Relation (14) entails that

$$\mathbf{U} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}})(\mathbf{V}^{\mathrm{T}})^{-1}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) \tag{16}$$

Post-multiplying (16) by the summation vector gives

$$\mathbf{Ue} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{Ve} \tag{17}$$

Substituting (15) and (17) in (1) yields

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x} + \mathbf{f} \tag{18}$$

From relations (18) and (3) it follows

$$\mathbf{f}^{*} \equiv \mathbf{f} \tag{19}$$

Substituting (16) in (2) yields

$$\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}})(\mathbf{V}^{\mathrm{T}})^{-1}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) + \mathbf{v}^{\mathrm{T}} \tag{20}$$

Post-multiplying (20) by $(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}}$ gives

$$\mathbf{e}^{\mathrm{T}}\mathbf{V}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}}) + \mathbf{v}^{\mathrm{T}}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}} \tag{21}$$

Substituting (15) in (21) we obtain[7]

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}} + \mathbf{v}^{\mathrm{T}}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}} \tag{22}$$

From relations (22) and (4) it follows

$$(\mathbf{v}^{*})^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}} \tag{23}$$

Hence, under the ITA, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $\mathbf{Ve} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{Ve} + \mathbf{f}$ and $(\mathbf{Ve})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}}) + \mathbf{v}^{\mathrm{T}}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}}$, respectively.

The By-Product Method

The by-product method (Stone, 1961, pp. 39-41) assumes that all 'secondary products' are 'by-products'[8] and that can be treated as negative inputs of the industries that they are actually produced. The Make matrix splits into $\widehat{\mathbf{V}}$ and $\mathbf{V}_2$, where $\widehat{\mathbf{V}}$ is the diagonal matrix that describes the 'primary products' of each industry and $\mathbf{V}_2$ is the off-diagonal matrix that describes the 'secondary products' of each industry. Thus, it holds

$$\mathbf{V} \equiv \widehat{\mathbf{V}} + \mathbf{V}_2 \tag{24}$$

In mathematical terms, the by-product method assumes that[9]

$$\mathbf{U} - \mathbf{V}_2 \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\widehat{\mathbf{V}} \tag{25}$$

and

$$\mathbf{x} \equiv \widehat{\mathbf{V}}\mathbf{e} \tag{26}$$

Relation (25) entails that

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv (\mathbf{U} - \mathbf{V}_2)(\widehat{\mathbf{V}})^{-1} \tag{27}$$

Combining (24) and (1) gives

$$\widehat{\mathbf{V}}\mathbf{e} \equiv (\mathbf{U} - \mathbf{V}_2)\mathbf{e} + \mathbf{f} \tag{28}$$

Substituting (25) and (26) in (28) yields

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U}, \mathbf{V})\mathbf{x} + \mathbf{f} \tag{29}$$

From relations (3) and (29) it follows

$$\mathbf{f}^* \equiv \mathbf{f} \tag{30}$$

Combining (24) and (2) gives

$$\mathbf{e}^{\mathrm{T}}\widehat{\mathbf{V}} \equiv \mathbf{e}^{\mathrm{T}}(\mathbf{U} - \mathbf{V}_2) + \mathbf{v}^{\mathrm{T}} \tag{31}$$

Substituting (25) in (31) yields

$$\mathbf{e}^{\mathrm{T}}\widehat{\mathbf{V}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U}, \mathbf{V})\widehat{\mathbf{V}} + \mathbf{v}^{\mathrm{T}} \tag{32}$$

Substituting (26) in (32) yields[10]

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U}, \mathbf{V})\hat{\mathbf{x}} + \mathbf{v}^{\mathrm{T}} \tag{33}$$

From relations (33) and (4) it follows

$$(\mathbf{v}^{\mathrm{T}})^* \equiv \mathbf{v}^{\mathrm{T}} \tag{34}$$

Thus, it follows that, under the by-product method, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $\widehat{\mathbf{V}}\mathbf{e} \equiv \mathbf{A}(\mathbf{U}, \mathbf{V})\widehat{\mathbf{V}}\mathbf{e} + \mathbf{f}$ and $(\widehat{\mathbf{V}}\mathbf{e})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U}, \mathbf{V})\widehat{\mathbf{V}} + \mathbf{v}^{\mathrm{T}}$, respectively.

Mixed Technology Assumptions

Mixed Technology Assumptions were suggested by Gigantes and Matuszweski (1968) and were incorporated in the 1968 System of National Accounts (United Nations, 1968, p. 50). This conversion method assumes that a part of the secondary products should be treated using the CTA and the remaining part should be treated using the ITA. The Make matrix splits into the matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ and it is assumed that $\mathbf{V}_1$ includes output that fits the CTA whilst $\mathbf{V}_2$ includes output that fits the ITA. Thus, it holds

$$\mathbf{V} \equiv \mathbf{V}_1 + \mathbf{V}_2 \tag{35}$$

Following Armstrong (1975), we define

$$\mathbf{A}_1 \equiv \mathbf{U}(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}})^{-1}(\widehat{\mathbf{V}_1^{\mathrm{T}}\mathbf{e}})(\mathbf{V}_1)^{-1} \tag{36}$$

and

$$\mathbf{A}_2 \equiv \mathbf{U}(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}})^{-1}\mathbf{V}_2^{\mathrm{T}}(\widehat{\mathbf{V}_2\mathbf{e}})^{-1} \tag{37}$$

where $\mathbf{A}_1$ is the direct requirements matrix for outputs included in $\mathbf{V}_1$, whilst $\mathbf{A}_2$ is the direct requirements matrix for outputs included in $\mathbf{V}_2$. Then, the commodity-by-commodity direct requirements matrix of the economy is defined as[11]

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv \mathbf{A}_1[\mathbf{I} - \mathbf{D}] + \mathbf{A}_2\mathbf{D} \tag{38}$$

where $\mathbf{D} \equiv (\widehat{\mathbf{V}_2\mathbf{e}})(\widehat{\mathbf{V}\mathbf{e}})^{-1}$. It can be seen that if $\mathbf{V}_2 = \mathbf{0}$, then $\mathbf{A} \equiv \mathbf{A}_1 \equiv \mathbf{U}\mathbf{V}^{-1}$, which is the CTA. On the other hand, if $\mathbf{V}_1 = \mathbf{0}$, then $\mathbf{A} \equiv \mathbf{A}_2 \equiv \mathbf{U}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}}(\widehat{\mathbf{V}\mathbf{e}})^{-1}$, which is the ITA.

On the basis of their critique to the ITA, ten Raa *et al.* (1984) constructed an alternative mixed technology model, assuming that $\mathbf{V}_2$ includes output that fits the by-product assumption. In mathematical terms, it is assumed that

$$\mathbf{V} \equiv \mathbf{V}_1 + \mathbf{V}_2 \tag{39}$$

$$\mathbf{x} \equiv \mathbf{V}_1\mathbf{e} \tag{40}$$

and

$$\mathbf{U} - \mathbf{V}_2 \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}_1 \tag{41}$$

which entails that

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv (\mathbf{U} - \mathbf{V}_2)\mathbf{V}_1^{-1} \tag{42}$$

Combining (39) and (1) gives

$$\mathbf{V}_1\mathbf{e} \equiv (\mathbf{U}-\mathbf{V}_2)\mathbf{e}+\mathbf{f} \qquad (43)$$

Substituting (40) and (41) in (43) yields

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x}+\mathbf{f} \qquad (44)$$

From relations (44) and (3) it follows

$$\mathbf{f}^* \equiv \mathbf{f} \qquad (45)$$

Combining (39) and (2) gives

$$\mathbf{e}^{\mathrm{T}}\mathbf{V}_1 \equiv \mathbf{e}^{\mathrm{T}}(\mathbf{U}-\mathbf{V}_2)+\mathbf{v}^{\mathrm{T}} \qquad (46)$$

Substituting (41) in (46) yields

$$\mathbf{e}^{\mathrm{T}}\mathbf{V}_1 \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}_1+\mathbf{v}^{\mathrm{T}} \qquad (47)$$

Post-multiplying (47) by $(\mathbf{V}_1)^{-1}(\widehat{\mathbf{V}_1\mathbf{e}})$ we obtain

$$\mathbf{e}^{\mathrm{T}}(\widehat{\mathbf{V}_1\mathbf{e}}) \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}_1\mathbf{e}})+\mathbf{v}^{\mathrm{T}}(\mathbf{V}_1)^{-1}(\widehat{\mathbf{V}_1\mathbf{e}}) \qquad (48)$$

Substituting (40) in (48) yields

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}}+\mathbf{v}^{\mathrm{T}}(\mathbf{V}_1)^{-1}\hat{\mathbf{x}} \qquad (49)$$

From relations (49), (40) and (4) it follows

$$(\mathbf{v}^*)^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}}\mathbf{V}_1^{-1}(\widehat{\mathbf{V}_1\mathbf{e}}) \qquad (50)$$

Thus, it follows that, under the mixed technology model introduced by ten Raa *et al.*(1984), the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $\mathbf{V}_1\mathbf{e} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}_1\mathbf{e}+\mathbf{f}$ and $(\mathbf{V}_1\mathbf{e})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}_1\mathbf{e}})+\mathbf{v}^{\mathrm{T}}\mathbf{V}_1^{-1}(\widehat{\mathbf{V}_1\mathbf{e}})$, respectively.

The Transfer Method

The Transfer method was proposed by Stone (1961, pp. 39-41) as an alternative method to treat 'by-products'. This method treats 'secondary products' as if they were bought by the industry where they are 'primary' and added to the output of that industry. In mathematical terms, it holds

$$\mathbf{x} \equiv (\mathbf{V} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} \tag{51}$$

whilst the direct requirements matrix derived from the transfer method is defined as[12]

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv (\mathbf{U} + \mathbf{V}_2^{\mathrm{T}})[(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) + (\widehat{\mathbf{V}\mathbf{e}}) - \widehat{\mathbf{V}}]^{-1} \tag{52}$$

where $\widehat{\mathbf{V}}$ is the diagonal matrix that describes the 'primary products' of each industry and $\mathbf{V}_2$ is the off-diagonal matrix that describes the 'secondary products' of each industry. Relation (52) entails that

$$\mathbf{U} + \mathbf{V}_2^{\mathrm{T}} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})[(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) + (\widehat{\mathbf{V}\mathbf{e}}) - \widehat{\mathbf{V}}] \tag{53}$$

Adding $\mathbf{V}_2^{\mathrm{T}}\mathbf{e}$ to both sides of (1) we obtain

$$(\mathbf{V} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} \equiv (\mathbf{U} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} + \mathbf{f} \tag{54}$$

Substituting (53) in (54) yields

$$(\mathbf{V} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})[(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) + (\widehat{\mathbf{V}\mathbf{e}}) - \widehat{\mathbf{V}}]\mathbf{e} + \mathbf{f}$$

or[13]

$$(\mathbf{V} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\mathbf{V} + \mathbf{V}_2^{\mathrm{T}})\mathbf{e} + \mathbf{f} \tag{55}$$

Substituting (51) in (55) gives

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x} + \mathbf{f} \tag{56}$$

From relations (56) and (3) it follows

$$\mathbf{f}^{*} \equiv \mathbf{f} \tag{57}$$

Adding $\mathbf{e}^{\mathrm{T}}\mathbf{V}_2{}^{\mathrm{T}}$ to both sides of (2) we obtain

$$\mathbf{e}^{\mathrm{T}}(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}}) \equiv \mathbf{e}^{\mathrm{T}}(\mathbf{U}+\mathbf{V}_2{}^{\mathrm{T}})+\mathbf{v}^{\mathrm{T}} \quad (58)$$

Substituting (53) in (58) yields

$$\mathbf{e}^{\mathrm{T}}(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}}) \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})[(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})+(\widehat{\mathbf{V}\mathbf{e}})-\widehat{\mathbf{V}}]+\mathbf{v}^{\mathrm{T}}$$

or

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}}+\mathbf{v}^{\mathrm{T}} \quad (59)$$

From relations (59) and (4) it follows

$$(\mathbf{v}^{*})^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}} \quad (60)$$

Hence, it follows that, under the transfer method, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations

$$(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}})\mathbf{e} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}})\mathbf{e}+\mathbf{f}$$

and

$$[(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}})\mathbf{e}]^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})[\widehat{(\mathbf{V}+\mathbf{V}_2{}^{\mathrm{T}})\mathbf{e}}]+\mathbf{v}^{\mathrm{T}}$$

respectively.

The ESA Method

The ESA (European System of Integrated Economic Accounts) method (Eurostat, 1979, pp. 116-7) recommends that ‘secondary products’ should be treated as if they were produced by the industries were these products are ‘primary’. Thus, it holds

$$\mathbf{x} \equiv \mathbf{V}\mathbf{e} \quad (61)$$

and the direct requirements matrix is defined as[14]

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv \mathbf{U}(\widehat{\mathbf{V}\mathbf{e}})^{-1} \quad (62)$$

which entails that

$$\mathbf{U} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}}) \tag{63}$$

Substituting (63) in (1) yields

$$\mathbf{Ve} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{Ve}+\mathbf{f} \tag{64}$$

Substituting (61) in (64) gives

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x}+\mathbf{f} \tag{65}$$

From relations (65) and (3) it follows

$$\mathbf{f}^{*} \equiv \mathbf{f} \tag{66}$$

Substituting (63) in (2) yields

$$\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}})+\mathbf{v}^{\mathrm{T}} \tag{67}$$

Adding $(\mathbf{Ve})^{\mathrm{T}}$ to both sides of (67) and after rearrangement we obtain

$$(\mathbf{Ve})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}})+\mathbf{v}^{\mathrm{T}}+(\mathbf{Ve})^{\mathrm{T}}-\mathbf{e}^{\mathrm{T}}\mathbf{V} \tag{68}$$

Substituting (61) in (68) yields

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}}+\mathbf{v}^{\mathrm{T}}+(\mathbf{Ve})^{\mathrm{T}}-\mathbf{e}^{\mathrm{T}}\mathbf{V} \tag{69}$$

From relations (69) and (4) it follows

$$(\mathbf{v}^{*})^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}}+(\mathbf{Ve})^{\mathrm{T}}-\mathbf{e}^{\mathrm{T}}\mathbf{V} \tag{70}$$

Consequently, with the use of the ESA method, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $\mathbf{Ve} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{Ve}+\mathbf{f}$ and $(\mathbf{Ve})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{Ve}})+\mathbf{v}^{\mathrm{T}}+(\mathbf{Ve})^{\mathrm{T}}-\mathbf{e}^{\mathrm{T}}\mathbf{V}$, respectively.

The Lump-Sum Method

The Lump-Sum (or Aggregation) Method (Office of Statistical Standards, 1974, p. 116) treats 'secondary products' as if they were produced as a 'primary product' of the industry that they are actually produced. Thus, it holds

$$\mathbf{x} \equiv \mathbf{V}^{\mathrm{T}}\mathbf{e} \tag{71}$$

and the direct requirements matrix is defined as[15]

$$\mathbf{A}(\mathbf{U},\mathbf{V}) \equiv \mathbf{U}(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}})^{-1} \tag{72}$$

which entails that

$$\mathbf{U} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}}) \tag{73}$$

Substituting (73) in (1) yields

$$\mathbf{Ve} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} + \mathbf{f} \tag{74}$$

Adding $\mathbf{V}^{\mathrm{T}}\mathbf{e}$ to both sides of (74) and after rearrangement we obtain

$$\mathbf{V}^{\mathrm{T}}\mathbf{e} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})(\mathbf{V}^{\mathrm{T}}\mathbf{e}) + \mathbf{f} + (\mathbf{V}^{\mathrm{T}} - \mathbf{V})\mathbf{e} \tag{75}$$

Substituting (71) in (75) yields

$$\mathbf{x} \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{x} + \mathbf{f} + (\mathbf{V}^{\mathrm{T}} - \mathbf{V})\mathbf{e} \tag{76}$$

From relations (76) and (3) it follows

$$\mathbf{f}^{*} \equiv \mathbf{f} + (\mathbf{V}^{\mathrm{T}} - \mathbf{V})\mathbf{e} \tag{77}$$

Substituting (73) in (2) gives

$$\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}}) + \mathbf{v}^{\mathrm{T}} \tag{78}$$

Combining (71) and (78) yields[16]

$$\mathbf{x}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\hat{\mathbf{x}} + \mathbf{v}^{\mathrm{T}} \tag{79}$$

From relations (79) and (4) it follows

$$(\mathbf{v}^{*})^{\mathrm{T}} \equiv \mathbf{v}^{\mathrm{T}} \tag{80}$$

Hence, under the Lump-Sum Method, the joint-product system described by the relations (1) and (2) is converted to the single-product system described by the relations $(\mathbf{V}^{\mathrm{T}}\mathbf{e}) \equiv \mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}^{\mathrm{T}}\mathbf{e} + \mathbf{f} + (\mathbf{V}^{\mathrm{T}} - \mathbf{V})\mathbf{e}$ and $(\mathbf{V}^{\mathrm{T}}\mathbf{e})^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{V}^{\mathrm{T}}\mathbf{e}}) + \mathbf{v}^{\mathrm{T}}$, respectively.

The Redefinition Method

The Redefinition method is used to move outputs and inputs of 'secondary products', that have distinctive production processes compared to those of the 'primary products' of each industry, to the industries where these products are 'primary'.[17] This method is most suitable to be applied for 'secondary products' that have production processes similar to the respective production processes of the industries where these products are 'primary'. Nevertheless, this method needs additional data on the production of the 'secondary products' that are not always available.[18]

EVALUATION OF THE CONVERSION METHODS

The next issue that comes up is which of the conversion methods is the most suitable for the problem at hand. Since there were not any objective criteria to test the consistency of the various methods, Jansen and ten Raa (1990) developed four desirable properties or, alternative, axioms that the various methods should fulfill. These properties are:

i. The 'Material Balance property':

$$\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V}\mathbf{e} = \mathbf{U}\mathbf{e} \tag{81}$$

This property implies that the requirements needed to produce the output should be equal to the actual inputs of the economy.

ii. The 'Financial Balance property':

$$\mathbf{e}^{\mathrm{T}}\mathbf{A}(\mathbf{U},\mathbf{V})\mathbf{V} = \mathbf{e}^{\mathrm{T}}\mathbf{U} \tag{82}$$

This property implies that, assuming that market prices are equal to 1,[19] the input cost of the output should be equal to the cost of the actual inputs.

iii. 'The Price Invariance property':

$$\mathbf{A}(\widehat{\mathbf{p}^{\mathrm{B}}}\mathbf{U},\widehat{\mathbf{p}^{\mathrm{B}}}\mathbf{V}) = \widehat{\mathbf{p}^{\mathrm{B}}}\mathbf{A}(\mathbf{U},\mathbf{V})(\widehat{\mathbf{p}^{\mathrm{B}}})^{-1}, \ \forall \mathbf{p}^{\mathrm{B}} > \mathbf{0} \tag{83}$$

where $\mathbf{p}^{\mathrm{B}}$ is the price vector relative to the base year prices. This property implies that whatever the base-year price is, the corresponding direct requirements matrix should be similar to the matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$.

iv. The ‘Scale Invariance property’:

$$\mathbf{A}(\mathbf{U}\hat{\mathbf{s}},\mathbf{V}\hat{\mathbf{s}}) = \mathbf{A}(\mathbf{U},\mathbf{V}), \ \forall \mathbf{s} > \mathbf{0} \tag{84}$$

This property guarantees that the direct requirements matrix does not depend on the activity levels of the economy.

Jansen and ten Raa (1990) proved that: (i) the CTA fulfils all the desirable properties; (ii) the ITA fulfils only the ‘Material Balance property’;[20] (iii) the By-Product method and the Mixed Technology model, constructed by ten Raa *et al.* (1984), fulfil the ‘Price’ and ‘Scale Invariance properties’; (iv) the Transfer method does not fulfil any of the properties; (v) the ESA method fulfils the ‘Material Balance’ and ‘Price Invariance properties’; and (vi) the Lump-Sum method fulfils only the ‘Scale Invariance property’.

Thus, it is concluded that only the CTA fulfils all the desirable properties. However, the CTA has been criticized because (i) it cannot be applied to the case of rectangular SUT;[21] and (ii) the direct requirements matrix that is derived from this method is possible to contain negative coefficients. Input-Output analysts have pointed out a number of reasons why negative coefficients may appear in the direct requirements matrix. These reasons are: (i) the same commodities may be produced with distinct technologies in different industries (in that case the CTA is not valid);[22] (ii) production classifications may be heterogeneous;[23] and (iii) SUT may have errors of measurement.[24, 25]

In order to overcome the problem of negative coefficients, analysts have proposed various procedures for removing the negative coefficients that may appear in the direct requirements matrix under the CTA.[26] A well known method is that proposed by Almon (1970, 2000), which consists of an iterative procedure of changes in the direct requirements matrix that converges to a (semi-)positive matrix.[27] Alternatively, Mixed Technology Models, *i.e.*, a combination of the CTA with another conversion method, are often used in order to overcome the problem of negative coefficients (see *e.g.*, Armstrong, 1975). Nevertheless, Mixed Technology Models cannot guarantee the derivation of a direct requirements matrix with non-negative coefficients.

On the basis of the previous analysis it can be said that none of the conversion methods can guarantee (i) consistency with the requirements of Input-Output analysis; and (ii) economically acceptable results.

We may find a way out of that problem by accepting that we live in a world where joint-product economic activities are common and by making use of general joint-product models inspired by von Neumann (1945) and Sraffa (1960). In the next section we present the essential ideas of the v. Neumann/Sraffa-based approach to the case of joint production as a preferable approach to treat actual joint-product tables.[28]

THE V. NEUMANN/SRAFFA-BASED APPROACH

A square linear system of joint production à la v. Neumann/Sraffa is defined by the pair $\{\mathbf{B},\mathbf{A}\}$, where $\mathbf{B}$ is the output matrix and $\mathbf{A}$ is the input matrix (both $\mathbf{B}$ and $\mathbf{A}$ are expressed in physical terms). Also, let $\mathbf{d}$ be the vector of final demand (in physical terms), $\mathbf{z}^{\mathrm{T}}$ be the row vector of value-added coefficients (in money terms), $\mathbf{y}$ be the vector of activity levels and $\mathbf{p}^{\mathrm{T}}$ be the row vector of market prices. Then we can write

$$\mathbf{By} \equiv \mathbf{Ay} + \mathbf{d} \tag{85}$$

and

$$\mathbf{p}^{\mathrm{T}}\mathbf{B} \equiv \mathbf{p}^{\mathrm{T}}\mathbf{A} + \mathbf{z}^{\mathrm{T}} \tag{86}$$

The above system is said to be strictly *viable* if it can produce a physical net surplus of any commodity. Formally,

$$\exists \mathbf{y} \geq \mathbf{0},\ (\mathbf{B}-\mathbf{A})\mathbf{y} > \mathbf{0} \tag{87}$$

A system $\{\mathbf{B},\mathbf{A}\}$ is said to be strictly *profitable* if there exists a price vector $\mathbf{p}$ for which all industries are profitable, *i.e.*,[29]

$$\exists \mathbf{p} \geq \mathbf{0},\ \mathbf{p}^{\mathrm{T}}(\mathbf{B}-\mathbf{A}) > \mathbf{0}^{\mathrm{T}} \tag{88}$$

A commodity i is said to be *separately producible* if it is possible to produce a net output consisting of a unit of that commodity alone with a nonnegative vector of activity levels. That is,

$$\exists \mathbf{y} \geq \mathbf{0},\ (\mathbf{B}-\mathbf{A})\mathbf{y} = \mathbf{e}_i \tag{89}$$

where $\mathbf{e}_i$ is a vector whose i th element is equal to 1 and all other elements are equal to zero.

A system $\{\mathbf{B},\mathbf{A}\}$ is said to be *all-productive* if all products are separately producible. Formally,

$$\forall \mathbf{d} \geq \mathbf{0},\ \exists \mathbf{y} \geq \mathbf{0},\ (\mathbf{B}-\mathbf{A})\mathbf{y} = \mathbf{d} \tag{90}$$

It can be easily seen that if $\{\mathbf{B},\mathbf{A}\}$ is all-productive then $(\mathbf{B}-\mathbf{A})^{-1} \geq \mathbf{0}$ (and *vice versa*).

A process, within a system $\{\mathbf{B},\mathbf{A}\}$, is called *indispensable* if it has to be activated whatever net output is to be produced. An all-productive system whose processes are all indispensable is called *all-engaging*. Formally, the system $\{\mathbf{B},\mathbf{A}\}$ is all-engaging iff the following two properties hold

$$\exists \mathbf{y} \geq \mathbf{0}, (\mathbf{B}-\mathbf{A})\mathbf{y} \geq \mathbf{0} \tag{91}$$

$$\{\exists \mathbf{y} \geq \mathbf{0}, (\mathbf{B}-\mathbf{A})\mathbf{y} \geq \mathbf{0}\} \Rightarrow \mathbf{y} > \mathbf{0} \tag{92}$$

It can be easily seen that if $\{\mathbf{B},\mathbf{A}\}$ is all-engaging then $(\mathbf{B}-\mathbf{A})^{-1} > \mathbf{0}$ (and *vice versa*). The concepts of 'all-productive' ('all-engaging') systems, introduced by Schefold (1971, pp. 34-5; 1978), are of significant importance since they correspond with systems that retain all the essential properties of decomposable (indecomposable) single-product systems.[30]

We now return to the actual economic system described by the Make and Use matrices, *i.e.*, the pair $\{\mathbf{V},\mathbf{U}\}$. The Make and Use matrices can be rewritten

$$\mathbf{V} \equiv (\hat{\mathbf{p}})\mathbf{B}(\hat{\mathbf{y}}) \tag{93}$$

and

$$\mathbf{U} \equiv (\hat{\mathbf{p}})\mathbf{A}(\hat{\mathbf{y}}) \tag{94}$$

Analogously, the vectors of final demand and value-added can be rewritten

$$\mathbf{f} \equiv (\hat{\mathbf{p}})\mathbf{d} \tag{95}$$

and
$$\mathbf{v}^{\mathrm{T}} \equiv \mathbf{z}^{\mathrm{T}}(\hat{\mathbf{y}}) \tag{96}$$

By setting the market prices equal to 1, *i.e.*,

$$\mathbf{p} = \mathbf{e} \tag{97}$$

the relations (93), (94) and (95) become $\mathbf{V} = \mathbf{B}(\hat{\mathbf{y}})$, $\mathbf{U} = \mathbf{A}(\hat{\mathbf{y}})$ and $\mathbf{f} = \mathbf{d}$, respectively. Thus, from relations (1), (93), (94), (95) and (97) we obtain

$$\mathbf{B}(\hat{\mathbf{y}})\mathbf{e} = \mathbf{A}(\hat{\mathbf{y}})\mathbf{e} + \mathbf{d} \tag{98}$$

or

$$\mathbf{By} = \mathbf{Ay} + \mathbf{d} \tag{99}$$

which is relation (85). Analogously, by setting $\mathbf{y} = \mathbf{e}$, it can be seen that there is a direct connection between relations (2) and (86). Consequently, the system described by the Make and Use matrices can be considered as the empirical counterpart of a joint-product system à la v. Neumann/Sraffa.[31] Namely, the Make matrix, $\mathbf{V}$, can be considered as the counterpart of the matrix $\mathbf{B}$, the Use matrix, $\mathbf{U}$, can be considered as the counterpart of the matrix $\mathbf{A}$, the vector of final demand, $\mathbf{f}$, can be considered as the counterpart of the vector $\mathbf{d}$ and the vector of value-added, $\mathbf{v}^{\mathrm{T}}$, can be considered as the counterpart of the vector $\mathbf{z}^{\mathrm{T}}$. In that case, an actual joint-product system will be said to be all-productive (all-engaging) when it holds $(\mathbf{V}-\mathbf{U})^{-1} \geq \mathbf{0}$ ($(\mathbf{V}-\mathbf{U})^{-1} > \mathbf{0}$).[32]

The conversion methods try to transform the joint-product system described by the pair of matrices $\{\mathbf{V},\mathbf{U}\}$ to the single-product system described by the pair of matrices $\{\mathbf{I},\mathbf{A}(\mathbf{U},\mathbf{V})\}$. This means that all conversion methods assume, implicitly or otherwise, that the there is a single-product system 'hidden', *i.e.* not directly observable, in the SUT. However, this is an unrealistic assumption.[33] On the other hand, the v. Neumann/Sraffa-based analysis of joint production constitutes a straightforward approach, *i.e.*, it does not rule out joint production, which is not based on any of the restrictive (and debatable) assumptions of the conversion methods.[34]

To sum up this section, given that (i) the pair $\{\mathbf{V}, \mathbf{U}\}$ can be considered as the empirical counterpart of the pair $\{\mathbf{B}, \mathbf{A}\}$; and (ii) joint production constitutes the empirical relevant case, it would seem reasonable that a straightforward treatment of actual joint-product tables (SUT), based on the v. Neumann/Sraffa-based analysis, to be preferred instead of trying to derive single-product tables (SIOT).

CONCLUDING REMARK

In this paper, alternative methods, used to convert Supply and Use Tables to Symmetric Input-Output Tables, have been reviewed. The evaluation of the conversion methods reveals that, despite the differences between them, they all rest on the assumption that there is a single-product system 'hidden' in the Supply and Use Tables that characterizes the economic structure of the real world. It has been argued that this is an unrealistic assumption and that a straightforward treatment of actual joint-product systems, on the basis of the v. Neumann/Sraffa-based analysis, constitutes a preferable approach.

Notes

1. It has to be noted that some of the 'joint' products that appear in the SUT may result from statistical classification. Therefore, these products do not correspond with the notion of joint production (see *e.g.*, Semmler, 1984, pp. 168-9 and United Nations, 1999, p. 77).

2. The on-diagonal elements of the Make matrix describe the so-called 'primary (or characteristic) product' of each industry and the off-diagonal elements describe the so-called 'secondary products'. The 'primary product' of an industry is defined as the output of that industry that comprises the primary source of revenues (Miller and Blair, 1985, p. 153).

3. In general, the SUT need not be 'square', *i.e.*, the number of commodities produced need not be equal to the producing industries (see *e.g.*, United Nations, 1999, p. 86 and Eurostat, 2008, p. 325).

Nevertheless, in what follows we assume, for simplicity's sake, that the Make and Use matrices are square.

4. The conversion methods that are presented in this paper convert the commodity-by-industry Make and Use matrices into a single-product system, where the direct requirements matrix is a commodity-by-commodity matrix. Nevertheless, there are corresponding methods that result in an industry-by-industry direct requirements matrix (see, *e.g.*, United Nations, 1968, pp. 48-50, Gigantes, 1970, Eurostat, 2008, ch. 11). Moreover, for an evaluation of these methods, see Rueda-Cantuche and ten Raa (2008). Even though our analysis could be extended to those conversions, we exclude them, for brevity's sake, from our analysis.

5. See, *e.g.*, van Rijckeghem (1967) and United Nations (1968, p. 49). The origins of this method can be found in Edmonston (1952, p. 567).

6. See, *e.g.*, United Nations (1968, pp. 49-50) and Schefold (1987, p. 1030). For a more analytical exposition of this method and a numerical example, see Miller and Blair (1985, pp. 166-9).

7. Note that $\mathbf{e}^{\mathrm{T}}\mathbf{V}(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}})^{-1}\mathbf{V}^{\mathrm{T}} \equiv \mathbf{e}^{\mathrm{T}}\mathbf{V}^{\mathrm{T}} \equiv (\mathbf{V}\mathbf{e})^{\mathrm{T}}$.

8. 'By-products' are products that are technologically linked to the production of the 'primary product' of the industry where it is actually produced (Stone, 1961, p. 39). The inputs needed for their production are considered to be 'low' in relation to the 'primary product' of the industry where they are produced (United Nations, 1999, p. 77, Viet, 1994, p. 33).

9. See, *e.g.*, ten Raa *et al.* (1984, p. 88) and Miller and Blair (1985, p. 173).

10. Note that $\mathbf{e}^{\mathrm{T}}\hat{\mathbf{V}} \equiv (\hat{\mathbf{V}}\mathbf{e})^{\mathrm{T}}$ and $\hat{\mathbf{V}} \equiv (\widehat{\hat{\mathbf{V}}\mathbf{e}})$.

11. For alternative ways of calculating the direct requirements matrix, using mixed technology assumptions, see Armstrong (1975, pp. 74-6) and Gigantes (1970, pp. 284-90).

12. See, *e.g.*, Jansen and ten Raa (1990, p. 215) and ten Raa and Rueda-Cantuche (2003, pp. 441-2).

13. Note that $(\widehat{\mathbf{e}^{\mathrm{T}}\mathbf{V}}) + (\widehat{\mathbf{V}\mathbf{e}}) - \hat{\mathbf{V}} \equiv \widehat{(\mathbf{V} + \mathbf{V}_2{}^{\mathrm{T}})\mathbf{e}}$.

14. See, *e.g.*, Viet (1994, pp. 38-40) and ten Raa and Rueda-Cantuche (2003, p. 443).

15. See, *e.g.*, Fukui and Seneta (1985, p. 177) and ten Raa and Rueda-Cantuche (2003, p. 444).

16. Note that $\mathbf{e}^{\mathrm{T}}\mathbf{V} \equiv (\mathbf{V}^{\mathrm{T}}\mathbf{e})^{\mathrm{T}}$.

17. See, *e.g.*, Viet (1994, p. 40) and United Nations (1999, p. 81).

18. See, *e.g.*, United Nations (1999, p. 81). For a presentation of the results that the Redefinition method yielded in the case of the USA Input-Output tables for the year 1992, see Guo *et al.* (2002, pp. 11-3).

19. That is to say, the physical unit of measurement of each commodity is that unit which is worth of a monetary unit (see, *e.g.*, Miller and Blair, 1985, p. 356).

20. ten Raa *et al.* (1984) showed that the direct requirements matrix derived under the ITA depends on the base year prices. In other words, they proved that the ITA does not fulfil the 'Price Invariance property'.

21. That is because this method requires the inversion of the Make matrix.

22. See, *e.g.*, Armstrong (1975, pp. 78-9).

23. See, *e.g.*, ten Raa *et al.* (1984, p. 93) and Rainer and Richter (1992).

24. See, *e.g.*, Armstrong (1975, p. 79). For a method to locate errors of measurement in the Make and Use matrices, see Steenge (1990).

25. ten Raa and van der Ploeg (1989) proposed a statistical model in order to interpret the source of negative coefficients. The application of their analysis to the case of the UK economy (for the year 1975) indicated that the source of negative coefficients cannot be attributed to errors of measurement in the SUT and that the CTA should be rejected (*ibid*., p. 6). On the other hand, Konijn and Steenge (1995) proposed an alternative way to construct Input-Output tables that relies on the concept of 'activity', defined as 'a set of production processes with input structures as homogeneous as possible' (*ibid*., p. 36), and introduced the so-called 'activity technology model', which is mathematically equivalent to the CTA. They argued that the direct requirements matrix, derived from the 'activity technology model', will contain negative elements only from causes such as heterogeneity or errors of measurement in the data.

26. For a detailed review of the available methods to remove negative coefficients, see ten Raa and Rueda-Cantuche (2005, pp. 4-13).

27. It has to be noted that convergence is guaranteed only if more than half of the production of each commodity is in its primary industry. Almon's method has been criticized for being without economic justification (see ten Raa *et al.*, 1984, p. 93 and ten Raa and Rueda-Cantuche, 2005, p. 10).

28. For a detailed exposition of the v. Neumann/Sraffa-based analysis and the connection between the works of v. Neumann and Sraffa, see Kurz and Salvadori (1995, ch. 8 and pp. 403-26; 2001).

29. In the case of joint production the conditions of viability and profitability are not equivalent (see Bidard, 1986, pp. 55-6). It need hardly be said that, in general, none of the usual laws of single-product systems holds true in the case of joint production (see Steedman, 1982; Bidard, 1997). Furthermore, for

an investigation of conditions under which the v. Neumann/Sraffa-based joint-product models lead to results consistent with the Marxian labour theory of value, see Semmler (1984, ch. 6).

30. See Schefold (1971, 1978, 1989) and Bidard (1996).

31. See also Flaschel (1980, pp. 120-1) and Bidard and Erreygers (1998, pp. 434-6).

32. It has been argued (Chilcote, 1997, Appendix A) that, in real economies, the off-diagonal elements of the Make matrix are 'small' and therefore joint production should not be overstated. Nevertheless, it has been found (see Mariolis and Soklis, 2009) that the actual systems that correspond with the SUT of the German (for the years 2000 and 2005) and Greek (for the years 1995 and 1999) economy are not 'all-productive'. Furthermore, empirical research from the author of this paper, based on the SUT of the Danish (for the years 2000 and 2004), German (for the years 1997-1999 and 2001-2005), Greek (for the years 1996-1998), Finnish (for the years 1995-2004), F.Y.R.O.M. (for the year 2005), Hungarian (for the years 2001-2004), Japanese (for the years 1970, 1975, 1980, 1985, 1990, 1995, 2000), Slovenian (for the years 2002-2005), Swedish (for the years 1995-2005) and USA (for the years 1998-2005) economy yielded the same result.

33. The awareness of joint-product processes has been already familiar to classical economists, such as Adam Smith (1904, book 1, ch. 11, § 56). For a review of the contributions of earlier economists to an analysis of joint production, see Kurz (1986). In addition, Jevons (1888, ch. 5, § 49) had pointed out that the cases of joint production form the general rule, to which it is difficult to find important exceptions. More recently, Steedman (1984) expressed a similar view (see also Faber *et al*., 1998).

34. For a similar view, see Lager (2007).

References

Almon, C. (1970) 'Investment in input-output models and the treatment of secondary products' in Carter, A. P. and Bródy, A. (eds) *Applications in Input-Output Analysis*, Amsterdam: North Holland.

Almon, C. (2000) 'Product-to-Product Tables via Product Technology with No Negative Flows', *Economic Systems Research* 12 (1): 27-43.

Armstrong, A.G. (1975) 'Technology Assumptions in the construction of United Kingdom input-output tables' in Allen, R. I. G. and Gossling, W. F. (eds) *Estimating and Projecting Input-Output Coefficients*, London: Input-Output Publishing.

Bidard, C. (1986) 'The maximum rate of profits in joint production', *Metroeconomica* 38 (1): 53-66.

Bidard, C. (1996) ‘All-engaging Systems’, *Economic Systems Research* 8 (4): 323-40.

Bidard, C. (1997) ‘Pure joint production’, *Cambridge Journal of Economics,* 21 (6): 685-701.

Bidard, C., and Erreygers, G. (1998) ‘Sraffa and Leontief on Joint Production’, *Review of Political Economy* 10 (4): 427-46.

Chilcote, E. B. (1997) *Interindustry Structure, Relative Prices, and Productivity: An Input-Output Study of the U.S. and O.E.C.D. Countries*, Ph.D. thesis, The New School for Social Research, mimeo.

Edmonston, J. H. (1952) ‘A Treatment of Multiple-Process Industries’, *The Quarterly Journal of Economics* 66 (4): 557-71.

Eurostat (1979) *European System of Integrated Economic Accounts (ESA),* 2nd ed., Luxembourg: Office for the Official Publications of the European Communities.

Eurostat (2008) *Eurostat Manual of Supply, Use and Input-Output Tables*, Luxembourg: Office for the Official Publications of the European Communities.

Faber, M., Proops, J. L. R. and Baumgärtner, S. (1998) ‘All production is joint production. A thermodynamic analysis’ in Faucheux, S., Gowdy,J. and Nicolaï, I. (eds) *Sustainability and Firms: Technological Change and the Changing Regulatory Environment*, Cheltenham: Edward Elgar.

Flaschel, P. (1980) ‘The Derivation and Comparison of Employment Multipliers and Labour Productivity Indexes using Monetary and Physical Input-Output Tables’, *Economics of Planning* 16 (3): 118-29.

Fukui, Y. and Seneta, E. (1985) ‘A theoretical approach to the conventional treatment of joint product in Input-Output Tables’, *Economics Letters* 18: 175-9.

Gigantes, T. (1970) ‘The representation of technology in input-output systems’ in Carter, A.P. and Bródy, A. (eds) *Contributions to Input-Output Analysis,* 270-90, Amsterdam: North-Holland.

Gigantes, T. and Matuszewski, T. (1968) 'Technology in input-output models', paper presented to the Fourth International Conference on Input-Output Techniques, Geneva.

Guo, J., Lawson, A. M. and Planting, M. A. (2002) 'From Make-Use to Symmetric I-O Tables: An Assessment of Alternative Technology Assumptions' Paper presented at the 14th International Input-Output Conference, 10-15 October 2002, Montreal Canada (http://www.bea.gov/papers/pdf/alttechassump.pdf).

Jansen, K. P. and ten Raa, T. (1990) 'The choice of model in the construction of input-output coefficients matrices', *International Economic Review* 31 (1): 213-27.

Jevons, W. S. [1871] (1888) *The Theory of Political Economy*, London: Macmillan and Co. Third Edition (http://www.econlib.org/LIBRARY/YPDBooks/Jevons/jvnPE1.html).

Konijn, P. J. A, and Steenge, A. E. (1995) 'Compilation of Input-Output Data from the National Accounts', *Economic Systems Research* 7 (1): 31-46.

Kurz, H. D. (1986) 'Classical and Early Neoclassical Economists on Joint Production', *Metroeconomica* 38 (1): 1-37.

Kurz, H. D. and Salvadori, N. (1995) *Theory of Production. A Long-Period Analysis*, Cambridge: Cambridge University Press.

Kurz, H. D. and Salvadori, N. (2001) 'Sraffa and von Neumann', *Review of Political Economy* 13 (2): 161-80.

Lager, C. (2007) 'Why and when are there negative coefficients in joint production systems with "commodity technology"', Paper presented at the 16th International Input-Output Conference, 2-6 July 2007, Istanbul, Turkey (http://www.iioa.org/pdf/16th%20Conf/Papers/LAGER.pdf).

Mariolis, T. and Soklis, G. (2009) 'Additive Labour Values and Prices of Production: Evidence from the Supply and Use Tables of the German and Greek Economy', *Working Papers in Input-Output Economics* WPIOX 09-002, International Input-Output Association.

Miller, R. E. and Blair, P. D. (1985) *Input-Output Analysis: foundations and extensions*, Englewood Cliffs: Prentice Hall.

Neumann J.von (1945-1946) 'A model of general economic equilibrium', *Review of Economic Studies* 13 (1): 1-9.

Office of Statistical Standards (1974) *Input-Output Tables for 1970*, Tokyo: Institute for Dissemination of Government Data.

Rainer, N. and Richter, J. (1992) 'Some Aspects of the Analytical Use of Descriptive Make and Absorption Tables', *Economic Systems Research* 4 (2): 159-72.

Rueda-Cantuche, J. M. and ten Raa, T. (2008) 'The choice of model in the construction of industry coefficients matrices', *Working Papers in Input-Output Economics* WPIOX 08-003, International Input-Output Association.

Schefold, B. (1971) *Mr. Sraffa on Joint Production*, Ph.D. thesis, University of Basle, mimeo.

Schefold, B. (1978) 'Multiple Product Techniques With Properties of Single Product Systems', *Journal of Economics* 38 (1-2): 29-53.

Schefold, B. (1987) 'Joint production in linear models' in Eatwell, J., Milgate, M. and Newman, P. (eds) *The New Palgrave: a dictionary of economics*, London: Macmillan.

Schefold, B. (1989) *Mr. Sraffa on Joint Production and Other Essays*, London: Unwin Hyman.

Semmler, W. (1984) *Competition, Monopoly, and Differential Profit Rates: On the Relevance of the Classical and Marxian Theories of Production Prices for Modern Industrial and Corporate Pricing*, New York: Columbia University Press.

Smith, A. [1776] (1904) *An Inquiry into the Nature and Causes of the Wealth of Nations*, London: Methuen and Co., Ltd., ed. Edwin Cannan. Fifth edition (http://www.econlib.org/LIBRARY/Smith/smWN.html).

Sraffa, P. (1960) *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*, Cambridge: Cambridge University Press.

Steedman, I. (1982) ‘Joint Production and the Wage-Rent Frontier’, *Economic Journal* 92 (2): pp. 377-85.

Steedman, I. (1984) ‘L’ importance Empirique de la Production Jointe’ in Bidard, C. (ed.) *La Production Jointe*, Paris: Economica.

Steenge, A. E. (1990) ‘The commodity technology revisited: theoretical basis and an application to error location in the make-use framework’, *Economic Modelling* 7 (4): 376-87.

Stone, R. (1961) *Input-Output and National Accounts*, Paris: OECD.

ten Raa, T. and van der Ploeg, R. (1989) ‘A statistical approach to the problem of negatives in input-output analysis’, *Economic Modelling* 6 (1): 2-19.

ten Raa, T. and Rueda-Cantuche, J. M. (2003) ‘The Construction of Input-Output Coefficients Matrices in an Axiomatic Context: Some Further Considerations’, *Economic Systems Research* 15 (4): 439-55.

ten Raa, T. and Rueda-Cantuche, J. M. (2005) ‘The problem of negatives in input-output analysis: a review of the solutions’, *Economic Working Papers at Centro de Estudios Andaluces* E2005/07, Centro de Estudios Andaluces.

ten Raa, T., Chakraborty, D. and Small, J. A. (1984) ‘An Alternative Treatment of Secondary Products in Input-Output Analysis’, *The Review of Economics and Statistics* 66 (1): 88-97.

United Nations (1968) *A System of National Accounts*, New York: United Nations, Series F, no. 2, rev. 3.

United Nations (1999) *Handbook of Input – Output Table. Compilation and Analysis. Studies in Methods. Handbook of National Accounting*, New York: United Nations, Department for Economic and Social Affairs, Statistics Division, Series F, No.74.

van Rijckeghem, W. (1967) ‘An Exact Method for Determining the Technology Matrix in a Situation with Secondary Products’, *The Review of Economics and Statistics* 49 (4): 607-8.

Viet, V. Q. (1994) ‘Practices in input-output table compilation’, *Regional Science and Urban Economics* 24 (1): 27-54.