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Last update: June 20, 2023

ECONOMETRICS
Problem Set 5
Due on: Tuesday, June 27, 2023.

Instructions: You may work alone or in teams of 2, but everyone must submit their own homework. The HW should be scanned in a pdf and emailed to my account `gkordas@panteion.gr` by 6pm of the due date.

This problem set is indented as an introduction to the issues researchers face when estimating *Simultaneous Equations Models* in econometrics. It is divided into two parts. Questions 1-3 concern a *recursive* form of the simple cobweb model of Supply and Demand. Questions 4-6 deal with a somewhat more complicated model which exhibits a genuine form of simultaneity. The file market1.dat contains the data for the first part, and the file market2.dat contains the data for the second part. Make sure that you use the correct dataset for each of the two parts of the problem set.

PART (I) Data file: market1.dat

Consider the model:

$$\begin{aligned} &\text{(Supply)} \qquad Q_t = \alpha_1 + \alpha_2 P_{t-1} + \alpha_3 Z_t + u_t \\ &\text{(Demand)} \qquad P_t = \beta_1 + \beta_2 Q_t + \beta_3 W_t + \upsilon_t \end{aligned}$$

Assume that u_t and υ_t are independent so the model is *recursive*. By "recursive" we mean that none of the explanatory variables in either equation are endogenous. To see this, note that in this model last period's price determines the current period's supply and then demand determines the current period's market clearing price. The variables W_t and Z_t may be regarded as strictly exogenous influences on supply and demand, respectively.

1. Estimate the model and illustrate its dynamic behavior by drawing a picture of the supply and demand functions for fixed values of the exogenous variables Z and W. Let $Z = Z_T, W = W_T$, i.e. fix them at their end-of-sample values.

Partial Answer: Since the system is recursive, it can be estimated using equation-by-equation OLS. Given the OLS estimates, evaluate the equations at Z_T and W_T to reduce them into a system of 2 equations with 2 unknowns. Graph the equations to obtain the usual Supply and Demand graph.

2. Make a point forecast of price for the next 6 periods assuming the exogenous variables remain fixed at their end-of-sample values. Suppose that the exogenous variables remained fixed at these values indefinitely; on average, what value would P take in equilibrium?

Partial Answer: Starting from the last period's price, P_T, and the last period's quantity, Q_T, iterate the system to find the equilibrium price and quantity P_E and Q_E, respectively. Graphing the iterations we obtain a cobweb-like picture that gives it's name to the model. It is instructive to see that the iteration converges *if and only if* the Demand is flatter than the Supply, i.e., if and only if $\alpha_2\beta_2 < 1$. Is this condition satisfied here?

3. Now suppose that the u_t 's are autocorrelated. Explain briefly why P_{t-1} can no longer be considered exogenous in this case. Devise a strategy for estimating the model, reestimate, and suggest a test of autocorrelation.

Partial Answer: The Demand equation is unaffected by the autocorrelation in the Supply equation, so nothing needs to be done there. Turning to the Supply equation we can write

$$\begin{aligned} Q_t &= \alpha_1 + \alpha_2 P_{t-1} + \alpha_3 Z_t + u_t \\ u_t &= \rho u_{t-1} + \varepsilon_t \end{aligned}$$

where ε_t is white noise. Then

$$\rho Q_{t-1} = \rho\alpha_1 + \rho\alpha_2 P_{t-2} + \rho\alpha_3 Z_{t-1} + \rho u_{t-1}$$

and subtracting from above we obtain,

$$Q_t = \rho Q_{t-1} + \alpha_1(1-\rho) + \alpha_2(P_{t-1} - \rho P_{t-2}) + \alpha_3(Z_t - \rho Z_{t-1}) + \varepsilon_t$$

This model has a nice iid error structure, but it is non-linear in its parameters. We will thus need to estimate it using *Nonlinear Least Squares* (NLS). The following code will do the job in `STATA` (All commands given here work in `STATA 7` but may fail in other versions, so if you have any other version make sure to begin your `.do` file with the command: `version 7`).

```
capture program drop nlg
program define nlg
if "`1'"=="?" {
global S_1 "rho a1 a2 a3"
global rho = .5
global a1 = .5
global a2 = .5
global a3 = .5
exit
}
replace `1'=$rho*LQ+$a1*(1-$rho)+$a2*(Lp-$rho*L2P)+$a3*(z-$rho*LZ)
end

gen time=_n
tsset time
gen LP = L.P
gen L2P = L2.P
gen LQ = L.Q
gen LZ = L.Z
drop if L2P == .
nl g Q
```

A more classical way of estimating the Supply model under autocorrelation of the errors is by the *Cochrane-Orcutt procedure*. This is a simple to implement iterative procedure and its details are discussed in p. 191 of Johnston and DiNardo. To estimate the Supply equation using the Cochrane-Orcutt procedure in `STATA` use the command

```
prais Q L.P Z, corc
```

and compare your results with the NLS estimates. The only thing that remains to be done is to devise a test for autocorrelation, but this is obvious now.

PART (II) Data file: market2.dat

Now consider the following simultaneous dynamic Supply and Demand model of the "cob-web" form:

$$\text{(Supply)} \qquad Q_t = \alpha_1 + \alpha_2 P_t + \alpha_3 P_{t-1} + \alpha_4 Z_t + u_t$$
$$\text{(Demand)} \qquad P_t = \beta_1 + \beta_2 Q_t + \beta_3 W_t + v_t$$

Now current period's price also influences current period Supply while current period Demand determines the equilibrium price.

4. Estimate the model both by OLS and 2SLS and compare your estimates. Interpret the differences.

> **Partial Answer:** Clearly, Q and P are endogenous in this model, while Z, W and P_{t-1} are exogenous. The STATA code for 2SLS is given by
>
> ```
> regress Q P LP Z (Z W LP)
> regress P Q W (Z W LP)
> ```
>
> where the variables in the parentheses are the instruments. Note that although the Demand equation has only one endogenous variable we use all the available instruments in both equations. The OLS estimates are very bad: the OLS estimate of the Demand equation has a very peculiar feature.

5. Test the hypothesis that the long run supply response to a change in the price is unity: i.e., that $\alpha_2 + \alpha_3 = 1$, and the hypothesis that the first period price effects are the same, i.e., $\alpha_2 = \alpha_3$.

> **Partial Answer:** You can use the `test` command in `STATA`, but make sure to explain what this `STATA` command does.

6. Suggest a test of the hypothesis that the errors in the two equations are uncorrelated. Reestimate to take account of this correlation if you find it, and compare your results with those in question 5.

> **Partial Answer:** I will not give any hints on how to test for correlation among the two errors – it is not difficult, just think about it. If you do this test correctly you will find that there is plenty of correlation, which suggests that the 2SLS estimates are consistent but not optimal. To get optimal estimates we should do 3SLS which is available in `STATA` using the following command
>
> ```
> reg3 (Q P LP Z) (P Q W), inst(Z W LP)
> ```
>
> Are the 3SLS estimates very different from the 2SLS ones?