The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria, I
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THE THEORY OF MARGINAL PRODUCTIVITY TESTED BY DATA FOR MANUFACTURING IN VICTORIA, I

SUMMARY

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I. The Statistical Measurement of Marginal Productivity.

In a study published three years ago,[1] one of the authors of this paper, in coöperation with Professor Charles W. Cobb, sought to devise an equation of production from statistics of fixed capital, labor, and physical productivity over a span of years in the manufacturing industries of the United States and of Massachusetts. It was found that a close approximation to the indexes of actual production could be obtained from the indexes of labor and of capital during these periods. By making the sum of the exponents of labor (L) and capital (C) equal to unity, or 1.0, a computed index of production (P') was obtained from the formula:

$$P' = bL^kC^{(1-k)} \tag{1}$$

1. Paul H. Douglas, The Theory of Wages.

This formula, as stated, was based on the assumption that production conformed to a simple homogeneous function of the first degree, namely, that an increase of 1.0 per cent in the quantities of both labor and capital would be accompanied by a corresponding increase of 1.0 per cent in the product. By the method of least squares, the values of the exponents for L and C were found to be .75 and .25 for manufacturing in the United States for the period 1899–1922, and .74 and .26 for the years 1890–1926 in Massachusetts.

Upon analysis it develops that the rate by which the marginal productivity of a given factor

$$\frac{\Delta P}{\Delta L} \text{ and } \frac{\Delta P}{\Delta C}$$

(represented by the symbols $M.P._L$ for labor and $M.P._C$ for capital) changes with each proportionate change in the quantity of that factor is (assuming that the quantity of the other factors remains constant) the sum of the exponents of the other factors of production.[2] This may be termed the coefficient of flexibility of the marginal productivity function of a factor: $\phi M.P._L$ and $\phi M.P._C$. It is obtained by dividing the relative change in the marginal productivity of a factor by the relative change in its quantity, and may be expressed as follows:

$$\phi M.P._L = \frac{\Delta M.P._L}{M.P._L} \Big/ \frac{\Delta L}{L} = \frac{\Delta M.P._L}{M.P._L} \cdot \frac{L}{\Delta L}, \text{ or} \tag{2}$$

$$\text{better} \qquad \frac{\Delta \log M.P._L}{\Delta \log L}$$

and similarly for $\phi M.P._C$.

The flexibility of the marginal productivity function for labor was, consequently, equal to -0.25 for the United States, and -0.26 for Massachusetts; and for capital -0.75, and -0.74, respectively. This meant that an increase of 1

2. For the mathematical development, see Section VII, and Mathematical Appendix, Section II. For further discussion, see Douglas, The Theory of Wages, pp. 150–155 and pp. 488–489. See also A. C. Pigou, The Economics of Welfare (2d ed. rev.; London, 1924), p. 623.

per cent in the quantity of labor, with capital constant, would cause the marginal productivity of labor to decrease by 0.25 or 0.26 per cent, while an increase of 1 per cent in the quantity of capital (labor constant) would cause the marginal productivity of capital to decrease by 0.75 or 0.74 per cent.

If we assume that the demand curve for each factor is identical with its marginal productivity curve, this would mean that the elasticity of demand for these factors (η_L and η_C) would be the reciprocal of their respective flexibilities, i.e.,

$$\frac{\Delta L \cdot M.P._L}{L \cdot \Delta M.P._L} \quad \text{or} \quad \frac{\Delta \log L}{\Delta \log M.P._L}$$

$$\text{and} \quad \frac{\Delta C \cdot M.P._C}{C \cdot \Delta M.P._C} \quad \text{or} \quad \frac{\Delta \log C}{\Delta \log M.P._C}. \tag{3}$$

These elasticities would be respectively -4.0 and -3.85 for labor, and -1.33 and -1.35 for capital. This meant that an increase of 1 per cent in the payment per unit of labor would normally be expected to occasion a decrease of approximately 4 per cent in the quantity of labor demanded; and a similar proportionate increase in the rate of interest would cause a decrease of $1\frac{1}{3}$ per cent in the amount of capital demanded.

It was also pointed out that, according to the marginal productivity theory, we should expect, under a state of perfect competition, that the share of the value produced by industry going to labor and capital would be identical with their relative exponents, namely, that labor would receive 75 and capital 25 per cent of the value product.[3] This, as a matter of fact, was found to be approximately the case; while in addition the movement of money and real earnings

3. These percentages were computed from manufacturing data alone, covering the years 1899 to 1922. For the period 1899 to 1916, an equation with exponents of 2/3 for labor and 1/3 for capital, gives results closer to actual manufacturing production in the United States. See, C. W. Cobb and P. H. Douglas, "A Theory of Production," American Economic Review Supplement, XVIII (March 1928), 159.

over periods of time proved to be closely similar to the relative movement of the marginal value productivity of labor as shown by the productivity formula.

Similar studies were also made for New South Wales for the period 1901–1927 by Mr. Aaron Director, with somewhat similar results, except that the exponents of L and C were found to be .65 and .35,[4] and that there was an apparently greater disparity between the actual share of the value product received by each factor, and that which we should expect from the production formula.[5]

Any such study obviously raises numerous queries, of which at least these two need to be considered. (1) Are the results accidental, or are they approximately confirmed by similar studies of other economies? (2) Is the production function which was chosen adequate to represent the real facts of economic life? For example, should the sum of the exponents be made equal to unity or should an opportunity be afforded for historical tendencies towards increasing or decreasing returns to be expressed in the equation? This would give a formula for this computed index production of:

$$P' = b\, L^k C^j \tag{4}$$

where the sum of k and j need not equal 1.0. If the sum of these exponents were greater than unity, then an allowance would be made for increasing returns. If they were less than unity, an allowance would be made for diminishing returns.[6] It is also a question whether we should treat the exponents as being constant throughout the period or should permit them to vary according to technological changes in industry

4. This was probably partially due to the fact that the capital index for New South Wales was lowered by taking into account replacements of depreciated capital at higher price levels.

5. This was partially caused by the fact that taxes and insurance costs were not deducted from value added in manufacture before computing the respective shares.

6. This has been suggested by Mr. David Durand in an article which will appear in a forthcoming issue of the Journal of Political Economy As we shall show later, under these circumstances the sum of (1) the marginal productivities of the various factors multiplied by (2) their respective quantities will not be equal to the total product.

and changes in the relative proportions of labor and capital.

The present authors have primarily sought to help meet the first of these queries by studying another economy for a somewhat similar period of time.[7] Sweden and the state of Victoria in Australia are obvious possibilities for further study, since they both publish excellent annual statistics on employment, production, value product, wages, prices[8] and the like. Since it is extremely difficult to obtain Swedish data on the total amounts of capital which have been employed, while such data, as we shall see, can be derived for Victoria, it was decided to confine the present study to that state and to reserve an analysis of Swedish data for a later time.

The period which we have chosen for study is that of the years 1907–1929. We have not gone back of 1907 because the Victorian and Australian statistics of physical production do not have adequate coverage before that time.[9] It was also thought best to terminate the period studied in 1929, and not to include the years of the great depression which were in so many respects abnormal.

It is perhaps proper to make a few preliminary comments on certain fundamental industrial differences between Victoria and the United States. The most striking of these is the fact that there is, on the average, appreciably less capital combined with a given unit of labor in Victoria in manufacturing than in this country. In 1914 only about a third as much capital per worker was used; in 1929 the average per worker was about half as much as in the United

7. We are indebted to Miss Yetta Abend for invaluable aid in computation and to Mr. Y. K. Wong for drawing the charts.

8. Altho extensive retail price data are gathered for Australian cost of living indexes, the data on the wholesale prices of manufactured and of semi-manufactured goods are incomplete. For example, the Melbourne Wholesale Price Index is heavily weighted with foodstuffs, includes imported commodities, and is not entirely representative of the prices of locally manufactured goods.

9. Data on quantity and value of product by manufacturing industries in Victoria were first published by the Commonwealth in 1907. Commonwealth Bureau of Census and Statistics, Production Bulletin: Summary of Australian Production Statistics, Nos. 2–23 (1907 through 1928–29), (Canberra; before 1926–27, Melbourne).

States.[1] Victoria depended on imports for many types of manufactured and semi-manufactured goods. A large share of manufacturing activity in Victoria was devoted to processing agricultural products, with the result that production statistics reflect droughts as well as the business cycle. The determination of wages by arbitration courts, protective tariffs, and bounties, all introduced artificialities into the situation which had their influence on the observed relationship between labor, capital, and production.[2]

We shall start our analysis with a description of the sources and methods which we have used in computing indexes of production (P), labor (L), and capital (C), for Victorian manufacturing during the period in question. We shall then proceed to (1) the determination of the production formula, (2) a comparison of how closely the production index (P') computed from (L) and (C) agrees with the actual product (P), (3) the measurement of the flexibilities of the marginal productivity functions for labor and capital and their corresponding elasticities of demand, (4) a comparison of the actual processes of distribution with the results we should expect from our formula. Finally, we shall compare our results with those formerly obtained, and consider in more detail some of the theoretical issues involved.

II. The Index of Production.

The index of production should measure changes in the physical volume of production, independent of the price level. For the present purpose, changes in the relative degree of fabrication are omitted, and it is assumed that the volume

1. These are rough estimates based on census totals, abstracting from differences in the price level and in the list of industries in the two countries If we compare ratios for one industry, choosing one in which the capital requirements are high relative to other industries, such as flour milling, we find that the average capital per worker in Victoria in 1919 was only one-fourth as much as in the United States. Estimates for 1929 are rough, since they are based only on data for horsepower per worker.

2. For data on economic conditions in Australia, see N. M. Windett, Australia as Producer and Trader, 1920–32 (London, 1933).

of production due to manufacturing is proportional to the physical volume of manufactured products.[3]

The physical volume of production can be measured either (a) by taking the aggregate sales value in successive years and then deflating the ratio thus obtained by an index of price change, or (b) by starting with the *physical production* of each kind of goods, expressing this total in relatives, and then weighting the separate ratios. The proportion of value added in manufacture furnishes a more satisfactory weight than prices, since it automatically corrects for the size of the unit in which price is expressed.[4] In this study, physical data were used as far as possible, and the deflated value of production was used only where no series measuring physical production were available. The formula adopted was:

$$\frac{\Sigma\left(\frac{q_1}{q_0}\right)v_0}{\Sigma v_0}, \tag{5}$$

where q_0 and q_1 represent the quantities of a manufactured commodity produced in the base year, and in a given year, and v_0 represents the percentage of the total value added in manufacture by the industry in question in the base year.

The annual reports by manufacturing establishments to the state government in Victoria and to the Commonwealth of Australia, in accordance with the Census and Statistics Act, furnish a valuable record of activity in the industries. Annual statistics are published in the Victorian Year Book,[5] The Official Year Book of the Commonwealth of Australia,[6] and in the Production Bulletins.[7]

3. Douglas, The Theory of Wages, p. 132.

4. For a more complex discussion of the issues involved in the construction of an index of physical production, see N A. Tolles and Paul H. Douglas, "A Measurement of British Industrial Production," Journal of Political Economy, XXXVIII (February, 1930), 2–8

5. Government Statist of Victoria, Victorian Year Book, Nos. 28–49 (1907 through 1928–29), (Melbourne, Australia). Manufacturing data published on a fiscal year basis beginning 1916–17

6. Commonwealth Bureau of Census and Statistics, Official Year Book of the Commonwealth of Australia, Nos. 1–22 (1908–29), (Canberra; before 1928, Melbourne).

7. Production Bulletin, Nos. 2–23 (1907 through 1928–29).

In 1902, the statisticians of the Australian States defined a factory as "an establishment employing on the average four persons or more, or an establishment employing less than four persons where machinery is worked by other than manual power, whether the business carried on is that of making or repairing for the trade (wholesale or retail) or for export." [8] At the same time, they also agreed on a standard classification of manufacturing industries into nineteen major groups.[9] This classification was maintained by the Commonwealth and by Victoria until the fiscal year 1930–31, when an extensive revision was made.[1] Under the nineteen major groups, there are numerous sub-groups of industries.[2] Altho the nineteen original groups remained the same, there was constant addition, consolidation and revision in the sub-groups of industries under each major group. In 1911 there were 86 such sub-groups; in 1928–29 no less than 91. In the case of approximately 30 industries, representing the bulk of manufacturing activity throughout the whole period 1907 to 1928–29, there was, however, relatively little alteration in the sub-groups.

The stability of the classification, and the fact that both the Commonwealth and the State follow the same classification, made it possible for us to use it as a framework in

8. Victorian Year Book, No. 49 (1928–29), p. 608.

9. The volume of manufacture was not evenly distributed between the groups. In the first seven groups,72 8 per cent of the value added in manufacture in 1911 was concentrated.

1. The initial classification differs from that used by the Census of Manufactures in the United States in that it groups under "Metals and Machinery Manufacture" industries which are shown in five separate groups in the United States Census classification. The Australian classification also provides separate groups for such small industries as "Surgical and Scientific Appliances," rather than including them in a miscellaneous category. Other differences are the result of the fact that Australian manufacturing consists to a large extent in processing agricultural commodities, or assembling imported parts, and even with the aid of the protective tariff and bounties does not have a full list of "secondary industries."

2. A major group, such as Clothing and Textiles, Fabrics, and Fibrous Materials, was composed of a number of sub-groups such as woolen and cotton mills, which in turn summarized the production of different types of woolen and cotton goods.

constructing the index. The procedure we have followed is analogous to that used in the case of the production index constructed by Day and Persons.[3] Thirty-seven individual series showing physical quantities of production in each of the various years and corresponding values of the product were obtained from the Production Bulletins.[4] The commodities covered are listed in Table 1.

These series cover only commodities produced, since quantities of raw materials consumed are reported without corresponding cost figures.[5] End products alone were included

3. E. E. Day and W. M. Persons, "An Index of the Physical Volume of Production," Review of Economic Statistics, II (1920), pp. 309–337, esp p. 310 and pp. 316–320.

4. Production Bulletin No. 2 (1907), Table 104, "Raw Materials Used and Commodities Produced in certain Industrial Establishments in the several States of the Commonwealth," pp. 65–66, and similar tables in succeeding issues to No. 23 (1928–29). This source was supplemented by figures for individual industries from Victorian Year Book, No. 28 (1907), pp. 709–722, and succeeding issues to No. 49 (1928–29). Neither of these sources quotes production data in physical terms for all industries in the industrial classification.

Using both of these sources, there were thirty-four series for which both quantity and value data were available. In addition to these, there were eleven series for which quantity data were given, but the value was included in a *joint value* figure for similar products. Three of these series were for tobacco products, and eight were for frozen and preserved meats. Because the quantity of meat preserved fluctuated so widely, and was so small relatively, it was decided to omit four series for preserved meat and an additional series for frozen poultry, thus reducing the number of these series from eight to three. Shawls and rugs were included as a single series in the original thirty-four, since the series was reported in that way in the Production Bulletins.

Three series, electricity, gas, and coke, were omitted since they were not considered typical of manufacturing activity as a whole. This reduced the number of physical production series from forty to thirty-seven.

5. For example, the quantity of greasy wool treated, and the quantity and value of scoured wool produced are reported under woolscouring works and fellmongeries. Under woolcombing and woolen and tweed mills, the quantity of scoured wool used in these mills is quoted, but the value of the materials used is not indicated, altho quantity and value of flannel, tweed and cloth produced are quoted. Materials such as greasy wool treated, and scoured wool used were not included in the index because it was difficult to determine weights without cost data. End products alone, such as scoured wool, flannel and tweed cloth produced, were included in the index.

TABLE 1. SERIES AND WEIGHTS[1] FOR PRODUCTION INDEX OF MANUFACTURED COMMODITIES, VICTORIA, AUSTRALIA

Group and Series	Per Cent of Group (1)	Per Cent of Total Value Added in Manufacture (1911) (2)
Total manufacturing industries represented		*100.0*
Class I: Treating animal and vegetable raw materials	*100.0*	*5.1*
Bone dust produced	6.1	.3
Scoured wool produced	21.5	1.1
Leather made	70.2	3.6
Basils made	2.2	.1
Class II: Oils and fats	*100.0*	*1.8*
Soap made	76.0	1.4
Candles made	24.0	.4
Class III: Stone, clay, glass, etc.	*100.0*	*5.3*
Bricks made	86.7	4.6
Lime produced	13.3	.7
Class IV: Working in wood	*100.0*	*2.4*
Forest saw mills	100.0	2.4
Class V: Metals and machinery	*100.0*	*25.8*
Deflated value series	100.0	25.8
Class VI: Food and drink, etc.	*100.0*	*29.5*
Bacon and ham	2.7	.8
Lard made	.2	.1
Butter made	11.8	3.5
Cheese made	.3	.1
Condensed milk made	.9	.3
Flour made	14.9	4.4
Bran and pollard	5.1	1.5
Jams and jellies	6.1	1.8
Sauce made	3.1	.9
Pickles made	1.4	.4
Aerated water	7.5	2.2
Cordial made	.8	.2
Malt produced	2.6	.8
Beer made	17.5	5.2
Spirits distilled	.7	.2
Cattle frozen	2.5	.7
Sheep frozen	3.6	1.1
Rabbits frozen	.4	.1
Tobacco	11.6	3.4

TABLE 1 (*Continued*)

Group and Series	Per Cent of Group (1)	Per Cent of Total Value Added in Manufacture (1911) (2)
Cigars made	4.5	1.3
Cigarettes made	1.8	.5
Class VII: Clothing, textile, fiber	*100.0*	*30.1*
Tweed and cloth	18.0	5.4
Flannel	24.6	7.4
Blankets	13.4	4.0
Shawls and rugs	.4	.1
Boots and shoes	42.9	12.9
Slippers	.6	.2
Uppers	.1	.1

WEIGHTS CLASSIFIED ACCORDING TO SIZE

	Number	Weights
Total — 38 series	*38*	*100.0*
Deflated series	1	25.8
Weights 5.0 or more	4	30.9
3.0 to 4.9	6	23.5
1.0 to 2.9	8	12.8
0.1 to 0.9	19	7.0

[1] Group weights based on value added in manufacture, 1911, for six groups plus Forest Saw Mills.
Weights for sub-groups within groups based on value added in manufacture, 1911.
Weights for series within sub-groups based on value of product, 1911.
Sources of original data: Victorian Year Book, No. 32 (1911), pp. 744–49; and Commonwealth Bureau of Census and Statistics Production Bulletin, No. 6 (1911), pp. 102–03.

in the index on the theory that they would represent activity in the individual industry, without the parallel series of materials used. This practice differs from Day and Persons' first index of production for the United States, in which they used both materials and products. For that index, figures on cost of materials were available.[6]

The industries for which quantity data in physical units are reported do not exactly coincide with the sub-groups of the industry classification, but usually do represent a large

6. Day and Persons, op. cit., pp. 311 and 321.

percentage of the product of a given sub-group.[7] Only six of the nineteen major groups of industries were represented by series with physical quantity data, namely, (1) Treating Animal and Vegetable Raw Materials (not otherwise classified). (2) Oils and Fats. (3) Stone, Clay and Glass, etc. (4) Working in Wood. (5) Food and Drink. (6) Clothing, Textiles and Fibrous Materials. These six major groups included 50.7 per cent of all the value added by manufacturing in 1911 and 52.0 per cent of such value in 1928–29.[8] The most important major group for which physical quantities of output could not be obtained was Metals and Machinery. The products of this group were so heterogeneous that they could not be reduced to comparable physical units. In 1911 it contained 17.6 per cent of the total value added in manufacture, in 1928–29 16.8 per cent. If this group is included with the other six, the percentage of net value product covered was 68.3 per cent in 1911, and 68.8 per cent in 1928–29. The total value of product of the Metals and Machinery group was, however, given for each of the years; and it was possible

7. For example, the number of yards of woolen tweed and cloth, and flannel, and the number of blankets, shawls and rugs were reported Coverage of the sub-group, "woolen and cotton mills," was not complete, however, since data on cotton cloth were not published. Furthermore, even when the physical series purported to represent the entire sub-group, the value of product quoted in the table showing physical production, sometimes showed slight discrepancies from the value of the sub-group in the industry classification table. In 1911, the sum of the values of boots, shoes, slippers and uppers was £1,845,000, while the total value of the sub-group, "boots, shoes and accessories," was £1,878,308. A sub-group was considered "represented" if a single series of physical data could be assigned to it, regardless of percentage coverage. Coverage was high, however, averaging 95.8 per cent for sub-groups represented by quantity data.

There were occasional discrepancies between items reported in the Victorian Year Books and the Production Bulletins. These appeared to result from differences in classification of individual firms. Value added in manufacture corresponding to the physical production series was not quoted, altho it was reported for sub-groups and for major groups in the industrial classification.

8. In the case of the Wood-Working group, the only industry for which we could obtain physical data was that for forest saw mills, which in 1911 had only 15 per cent of the value of product in the Wood group. We gave to the Wood group, therefore, only the importance of forest saw mills.

to deflate this by a specially constructed index of production costs. This index was composed of (a) a wholesale price index of metals and (b) an index of money wage rates for adult males in the engineering and metals trades.[9] Prices were weighted 57.8 per cent and wage rates 42.2 per cent, in accordance with the proportion of wages and raw materials costs in the base year of 1911.[1]

The index of Metals and Machinery production derived from the deflated value figures may suffer from overcorrection in war years, since the index for 1917–18 falls to 53.3 per cent of 1911 and is only half what it was in 1914. This might result if the price component of the deflator was not entirely representative of raw material costs in the industry. Efforts to introduce a correction for changes in productivity per worker during the period were not entirely satisfactory, and no adjustment of this type was made in the final deflated values. It was concluded that it was better to use the deflated values, even tho they were subject to these limitations, than to omit this important series from the index.

We have seen that the seven major groups of industries for which we have actual and deflated quantity data turned out slightly over two-thirds of the value added in manufacturing in Victoria. How representative, then, are the specific industries within these major groups for which we have data? There are thirty-eight[2] of these industries which

9. Data for prices and wages from Commonwealth Bureau of Census and Statistics Labour Reports, Nos. 1–20 (1912–29), (Canberra; before 1928, Melbourne). The prices used were the Melbourne wholesale prices of metals. Since some of the commodities quoted were largely imported, and imports were almost nonexistent during the war, these items were omitted in choosing prices for the index The commodities included were pig iron, rods and bars, tinned plates, copper, zinc, lead sheet and piping. Weights used were those of the Melbourne Wholesale Price Index, based on the average quantities sold, 1906 to 1910. Ibid., No. 1 (1912), p. 20.

1. The values of production for the metal series, the deflation index of costs, and the physical index derived from the deflated values are shown in Table I of a mimeographed supplement which will be supplied on request.

2. This includes thirty-seven series of physical quantity data and

can in turn be classified into twenty-eight sub-groups. These thirty-eight industries produced in 1911 52.6 per cent of all the value product of manufacturing.[3] Should they be weighted strictly according to their relative proportions of this percentage, or should they be taken as representative of larger sub-groups and major groups? Since they were responsible for 97 per cent[4] of all the gross value produced in 1911 by their respective sub-groups, they can certainly be taken as representative of the latter.

The value of the product of these twenty-eight represented sub-groups was responsible, in turn, for 70.8 per cent of the total value of output, and for 68.5 per cent of the value added in manufacturing in the seven major groups as a whole. In view of this coverage, the computed index may also be taken as representative of these larger groups. Using the total value of product as a standard, these seven major groups cover 76.7 per cent of the gross sales value of product in all manufacturing industries. In terms of value added in manufacture, the coverage is 68.3[5] per cent in 1911 and 68.8 per cent in 1928–29.[6] If employment were used to measure

one series of deflated data representing the entire Metals and Machinery group. In 1911 this latter group was composed of nine sub-groups, all of which have been considered as represented sub-groups and included in the total of twenty-eight sub-groups.

3. This percentage is based on the value of product corresponding to each of the thirty-seven series of physical production data, plus the value of product of the Metals and Machinery group, all expressed as a ratio to the total value of product in all manufacturing industries in 1911. For coverage ratios see Table II of the mimeographed supplement referred to above.

4. The percentage of the value of product in the thirty-eight series to the total value of product in the twenty-eight sub-groups.

5. This differs from the percentage quoted on p. 8 because of the reduction in the coverage of the Wood group to correspond to forest saw mills only. See Table III of mimeographed supplement for a summary of percentages of value added in manufacture covered by the production index, in terms of groups and sub-groups.

6. The coverage is comparable with that obtained by Day and Persons, op. cit., pp. 309 and 321. In their indexes based on census data, the ratios of value added in manufacture by represented sub-groups, to group value added in manufacture, in 1909, varied from 14 per cent for Paper Products to 98 per cent for Tobacco. A similar ratio for the twelve represented groups to the fourteen groups reported

coverage, the percentage would be 73.4 per cent in 1911 and 72.6 per cent in 1928–29.

The year 1911 was adopted as the base for the production relatives, since that had been chosen as the base of the Melbourne Wholesale Price Index,[7] and had been considered a typical pre-war year.

It is interesting to note that the relative percentages of value added in manufacturing by the various groups of industries did not change appreciably over the period covered. The maximum change in absolute terms between 1911 and 1928–29 was, for example, 3 per cent,[8] and for most groups was closely similar. It followed, therefore, that very little difference would be caused by using the end-year values as weights, i.e.

$$\frac{\Sigma\left(\frac{q_0}{q_1}\right)v_1}{\Sigma v_1} \tag{6}$$

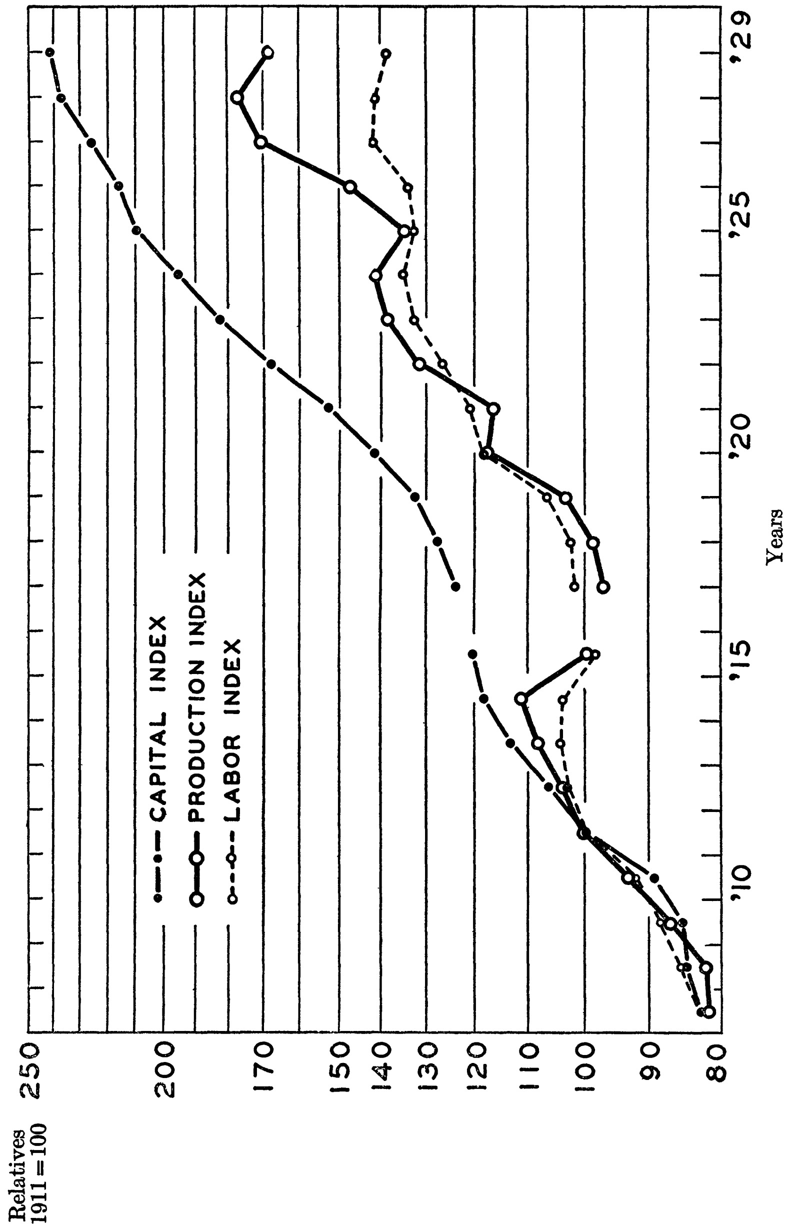
and that the system of base-year weights used would be as satisfactory as either this or Fisher's geometrical average of the two sets of index numbers. A system of weights was, therefore, developed which used the value added in manufacturing in 1911 by the represented major groups as a

by the United States Census denotes 89 per cent coverage in terms of value added in manufacture. Day and Persons state that the ratio of value added by manufacture ascribable to products actually covered, to the total value added by manufacture for the entire fourteen major groups is estimated to be 40 to 45 per cent.

7. Labour Report, No. 1 (1912), pp. 43–66. An additional reason for the selection was that 1911 had also been used as the base of the Production Index for New South Wales (Douglas, The Theory of Wages, pp. 167–172) and it was desired to make these two indexes comparable. It is of interest to note that Day's Index of Production in the United States used 1909 as a base.

8. This is in the Textile group, which had a proportion of 30.1 in 1911, as compared with 33.1 per cent in 1928–29. This was caused by relative increases in the sub-groups of (1) tweed and cloth and (2) boots and shoes. Flannel decreased in relative importance. Similar stability is shown in the percentages for the numbers employed in the different groups. There was a slightly greater change in the group proportions as measured by the number of establishments. Differences in relative amounts of labor and capital affected the proportion of employment compared to the proportion of value added in manufacture. In 1911,

Chart 1. — Indexes of Fixed Capital, Labor and Production, Manufacturing Industries, Victoria



framework.[9] Within this framework, weights for individual industries were determined from the value added in manufacture by the represented sub-groups of industries. When a sub-group contained more than one industry, weights were determined on the basis of the proportion of the gross value of product of the industry to the total gross value for reported series in the sub-group.[1] The weights determined from 1911 data are presented in Table 1, with a frequency distribution of these weights. Altho there were nineteen series with weights less than 1.0 in the final index, all such series were included in the interest of accuracy, since collectively they covered 7.0 per cent of the index.

Separate indexes of production were constructed for each of the represented groups, so that it would be possible to study changes in production by type of industry as well as in total. The combined index of production is shown with the indexes of employment and capital in Table 2 and Chart 1.[2]

the Food group claimed 20.1 per cent of the value added in manufacture and 12.9 per cent of employment, while Clothing and Textiles had 20.5 per cent of the value added in manufacture and 35 5 per cent of employment. The relative proportions were virtually unchanged in 1928–29.

9. Since 69.8 per cent of the total value of reported series (i.e., for commodities with quantity and value data) was in the Food group, it is obvious that weights based on the values of individual series as they were reported would be fortuitous and unrepresentative. Weights based on group totals were consistent with assumptions as to coverage.

1. A similar procedure was followed by Day and Persons in the determination of weights for a Production Index for the United States, op. cit., pp. 310–11.

In the cases of flour milling, jams, pickles and preserves, textiles, and boots and shoes, the weights of individual series within the group were adjusted slightly to permit these series to bear the weight of industries whose products were closely related to the reported series Estimates also had to be made as to the relative weights to be assigned to series in the sub-groups covering preserved and frozen meat and tobacco. An office memorandum gives the details of this adjustment

2. Examination of individual production series shows that the low level of the Production Index between 1915 and 1918–19 was largely attributable to the index of the deflated Metals and Machinery values, altho Stone, Clay and Glass, and Forest Saw Mills also contributed to this result. An index constructed from the physical quantity data alone (five groups plus Forest Saw Mills) declined from 112.6 in 1914 to 101.5 in 1915, and rose to 120.1 by 1918–19. The index for 1920–21 was at

TABLE 2. — INDEXES OF LABOR, CAPITAL, AND PRODUCTION IN MANUFACTURING INDUSTRIES, VICTORIA, AUSTRALIA, JANUARY, 1907–JUNE, 1929 [1]
(1911 = 100)

Year	Index of Labor[2] (1)	Index of Capital[2] (2)	Index of Production[2] (3)
1907	82.8	82.7	81.6
1908	85.5	84.6	82.2
1909	88.5	85.5	87.0
1910	92.0	89.2	93.1
1911	100.0	100.0	100.0
1912	103.0	106.3	103.5
1913	104.2	112.9	108.5
1914	103.6	118.3	110.9
1915	98.2	120.1	99.1
1916–17	101.6	123.6	96.9
1917–18	102.4	128.2	98.8
1918–19	106.4	132.9	103.4
1919–20	118.7	141.7	117.5
1920–21	121.0	153.1	116.4
1921–22	126.4	168.5	128.7
1922–23	133.0	182.1	138.1
1923–24	134.9	195.6	141.2
1924–25	132.6	210.2	134.2
1925–26	133.8	215.9	147.3
1926–27	141.9	226.6	170.6
1927–28	141.1	237.5	177.0
1928–29	138.3	241.5	168.6

[1] Beginning with 1916–17, data reported by census years, July 1 to June 30. Because of this change, data for first half of 1916 are not available.

[2] Indexes carried to two decimal places in subsequent computations.

Sources of original data: Commonwealth Bureau of Census and Statistics, Production Bulletins, Nos. 2–23 (1907 through 1928–29); Victorian Year Books, Nos. 28–49 (1907 through 1928–29).

In general terms, this shows an increase from 81.6 in 1907 to 168.6 in 1929, or more than a doubling. There was an advance of 29 points, or 36 per cent, between 1907 and 1914. During the war time, however, there was an appreciable

123.5 as compared with 135.2 in 1919–20. This index, as well as the production index based on six groups plus forest saw mills, shows a sharp upward trend.

decrease. While a part of this drop is undoubtedly due to the movement of men into the army and to shipping disturbances which interfered with the flow of English products for further fabrication, it is also probable that our price index for metals caused a too severe deflation of that series, and hence for production as a whole. After the war the index rose to 141.2 in 1923–24. After the relapse in 1924–25, it rose again to 177.0 in 1927–28, from which there was a fall of 8 points, or 4.7 per cent, in the final year. From 1919–20 to 1920–21 there was only a slight decline — from 117.5 to 116.4. The industries processing agricultural products did, to be sure, fall more sharply during these years, but this was largely offset by other industries which showed gains. These may have resulted in part from the high protective Tariff Act of 1920, which distinctly encouraged Australian manufacturing. Since these movements of our index of production did not seem inconsistent with other information on business conditions in Australia,[3] it was adopted as one of the factors of the production equation.

III. The Index of Labor.

As the measure of labor, the average number of persons employed was chosen. It was assumed that the productive powers of the average worker were constant over the period and that the number of hours worked per week was constant. In practice these two factors would probably have helped to offset each other, since the former (with the exception of the war period) was probably rising while the latter was falling. It is obvious also that an index of the numbers employed is not as sensitive an index of cyclical fluctuations as is one of man-hours worked, since it does not allow for over-time hours worked during the periods of prosperity or for short-time during depression.[4] Since no allowance is

3. See below, pp. 30–32.

4. Prof. J. M. Clark, however, considers employment data preferable to man-hour data for the construction of the labor index, because they minimize the distortion in proportions of capital to labor which results from combining a relatively stable capital index (containing unused capacity in periods of depression) with a cyclically fluctuating index

made for the trend toward the shorter week, the equation, by somewhat overstating the physical quantities of labor, may tend to give capital too little credit for increased output.[5]

The statistics on the total number employed include working employers, managers and overseers, accountants, clerks, engineers, carters, messengers, and skilled and unskilled workers. During the period January, 1907, through June, 1928, the average annual employment, as quoted in the Victorian Year Book, is adjusted on the basis of the number of months of operation when this was less than twelve. The method of arriving at this average was changed in 1928–29. Thereafter, the average was taken for the whole year. There are no revised figures published for the preceding year to make it possible to determine the probable effect on 1928–29 data of this revision. The drop in employment between 1927–28 and 1928–29 is also accompanied by a decline in the production index, so it is assumed that the decrease in number reported employed is not entirely the result of the revision in the method of calculation.[6]

In our choice of the industries to be covered by the employment index, we sought to make the coverage as close as possible to that of the production index. We therefore chose the total numbers employed in each of the years in the seven major groups of industries for which there were production data. These formed 73.4 per cent of all those employed in 1911 and 72.6 per cent of those in 1928–29.[7] The total numbers employed in these groups were then reduced to relatives by dividing the total for each year by that for 1911. This is shown in column 1 of Table 2. It increases from a relative of 83 in 1907 to 142 in 1926–27, or

of labor. J. M. Clark, "Inductive Evidence on Marginal Productivity," American Economic Review, XVIII (September, 1928), 453–455.

5. Ibid., pp. 454–455.

6. Victorian Year Book, No. 49 (1928–29), p. 637.

7. We experimented also with an index based on employment in the sub-groups represented by production data, but found that an implicitly weighted aggregate was biassed, and that different proportions between labor and capital in individual industries made it difficult to choose group weights to construct an index from relatives of employment in represented sub-groups.

a rise of almost 60 points (in terms of 1911 as 100). In 1928–29 the value was 138.

A comparison of the index of labor with that for production (see Table 2 and Chart 1) shows that in general it lies below the production index, as was the case in previous studies,[8] but that for the period July, 1916, through June, 1921, the production index fell below the employment index. This suggests a relative inefficiency of labor during the war and immediate post-war period, which may have been caused by the substitution of less skilled workers for those who entered the armed forces, as well as by the general disorganization caused by shipping and other difficulties.

IV. The Index of Capital.

In computing the index of capital we adopted the following procedure:

(1) We were compelled to omit all consideration of working capital, for which no annual statistics were given.

(2) We excluded land values, both because the quantity of land probably did not vary greatly from year to year, and because it is difficult to measure a unit of "land." [9]

(3) We computed from data in the Australian Production Bulletins annual values for (a) buildings and fixtures,[1] and

8. United States, Massachusetts and New South Wales, Douglas, The Theory of Wages, pp. 144, 160, and 168.

9. For further discussion of reasons for omitting land from the index of capital, see J. M. Clark, op. cit., pp. 456–457.

In order to omit land values from the index, it was necessary to estimate the proportion of land values in the reported joint total value of land and buildings. The proportion was estimated at 33 per cent. The method of arriving at this estimate is described in an office memorandum which will be loaned to any inquirer. In brief, six Victorian industries for which separate land values were reported, showed that land values were 32.1 per cent of the total value of land and buildings. A similar percentage (33 per cent) was derived from a study by the Missouri Bureau of Labor Statistics, [Missouri Bureau of Labor Statistics, Forty-fourth Annual Report for Year ending December 31, 1923 (Jefferson City, 1924), p. 144], which quoted value of land, and a combined value for buildings and machinery. Estimates given in Douglas, The Theory of Wages, p. 115, for the United States in 1922 were used to obtain a separate percentage for building values.

1. In accordance with the estimate described above, these were

(b) plant and machinery, for the seven major groups of industries in Victoria covered in the production and employment indexes. In other words, the capital index was made comparable with these.

(4) Annual increments of investment in terms of the price level of the given year were found by subtracting the total for the preceding year from that of the current year.

(5) The increments in the value of buildings and fixtures were then deflated by an index of building costs. This index was composed of (a) the Melbourne Wholesale Price Index of Building Materials [2] (which, however, did not contain any structural steel), (b) the Melbourne Wholesale Price Index of Metals [3] with coal excluded (to represent structural steel) and (c) an index of the average weekly wage rate of adult males in the building trades in the principal cities of Australia.[4] Since Victorian statistics of capital were shifted from a calendar to a fiscal year basis beginning with 1916-17, two-year moving averages of these component series were taken beginning with that time, in order to make the time periods comparable. In the construction of this deflation index, the first of these series was given a weight of 56, the second of 14, and the third of 30. By dividing the annual increments in building values by this index of costs in terms of 1907, we obtained deflated increments in terms of the prices of that year. When these are cumulated from the 1907 value, we obtain a series for capital invested in buildings (in terms of 1907 costs).

(6) The annual increments for plant and machinery were deflated by a combined cost index composed of (a) the Melbourne Wholesale Price Index of Metals [5] (excluding coal) and (b) an index of the average weekly wage rates in

obtained by taking 67 per cent of the total value reported for land and buildings.

2. Labour Reports, Nos. 1–20 (1912–29).

3. Ibid., and J. L. K. Gifford, Economic Statistics for Australian Arbitration Courts, Melbourne University Economic Series, No. 3 (Melbourne, 1928), pp. 104–108.

4. Labour Reports, Nos. 1–20 (1912–29).

5. Ibid., and Gifford, op. cit., pp. 104–108.

the engineering trades.[6] Two-year moving averages of these indexes were taken beginning in 1916–17. In accordance with the relative percentage of raw material and labor costs in 1911, the former was given a weight of 58 and the latter one of 42.

(7) The deflated annual totals in terms of 1907 costs for buildings and fixtures and for machinery, implements and tools were then added to obtain a total for each year. These totals were then put into relatives in terms of 1911 (column 2 of Table 2).[7] The resulting index shows an approximate trebling during the period in question: from a relative of 83 in 1907 to 242 in 1928–29.

It is probable that this index somewhat overstates the actual growth of total capital in manufacturing during this period. (1) It does not include working capital, which probably grows more slowly than fixed capital and probably at a rate somewhere between that for production and fixed capital.[8] (2) Altho the statistics of capital value do apparently make some allowance for depreciation,[9] it is possible that they do not fully do so. In this event, some of the apparent increases in the later years were in all probability not net additions to capital, but were instead replacements, at higher cost levels, of equipment originally installed at lower price levels. Had it been possible to consider these factors, there might well have been a smaller increase in the capital index, which in consequence would have been closer to the index of production, and hence have led to a larger exponent being given to C in the production equation.

Wassily Leontief, in his able article, "Interest on Capital

6. Labour Reports, Nos. 1–20 (1912–29).

7. A supplementary table showing the component series of the deflation indexes and the methods used in constructing the capital index has been mimeographed and will be supplied on request. (Table IV of the supplementary tables)

8. For further discussion of effect of exclusion of working capital from capital index, see Clark, op. cit., pp. 456–458, and Douglas, The Theory of Wages, pp 207–209.

9. Victorian Year Book, No. 55 (1934–35), p. 504, and Federal Commissioner of Taxation, Commonwealth of Australia, Income Tax Order No. 1196 (Melbourne, 1928) and No. 1217 (Canberra, 1936).

and Distribution, a Problem in the Theory of Marginal Productivity,"[1] has raised the question of adding two terms to the production equation, which provide for the turnover time of the wage fund and the turnover time for all the other capital parts, and include interest for these. The depreciation of our capital index, in accordance with an estimated average turnover rate, would seem to make the second term less necessary. Since we were unable to secure data on working capital, the first term suggested by Professor Leontief could not be included. The probable effects of this omission are discussed later.[2]

V. The Equation of Production.

The indexes for production, capital and labor are shown on a semi-logarithmic scale in Chart 1. Values are quoted in Table 2. The intermediate position of the production index, between those for capital and labor, is similar to that shown by indexes for the United States, Massachusetts and New South Wales. There is an exception to this rule, however, during the war and immediate post-war years, when the rate of growth of the employment index was faster than that of the production index.

As has been stated, it was assumed that there was a constant or "normal" relationship between production and the factors capital and labor, and that this could be expressed by the equation:

$$P'(L,C)=bL^{k}C^{(1-k)}$$

where P' is the computed index of production. This is equivalent to assuming that the function $P(L, C)$ is by definition a homogeneous function of the first order. This implies that if labor and capital were increased by a factor "m," product would be increased in the same proportion. It also assumes that changes in production are the resultants of changes in the quantities of labor and fixed capital alone, since land and working capital are omitted, and that the

1. Quarterly Journal of Economics, xlix (1934–35), 147–163.
2. See the succeeding installment of this article, Section IX.

productive power of an average laborer and of a unit of capital (i.e., a dollar of constant purchasing power of capital goods) was constant from year to year.[3]

Assuming this relationship between the variables, the next step was to determine numerical values for b and k, such that the sums of the squares of the deviations of P' (i.e., the computed product) from P were a minimum.[4] Using the indexes of production, labor and capital for Victoria, the value of b was found to be .97 and the value of k, .71. This gave the following production function:

$$P' = .97 L^{.71} C^{.29}.$$

The value of b was relatively unimportant, serving only to reduce the computed amount of product slightly below what it would have been otherwise.[5] This meant that with capital constant, an increase or decrease of 1.0 per cent in the quantity of labor would normally be accompanied by an increase or decrease of 0.71 per cent in the volume of production. Similarly, a change of 1.0 per cent in the quantity of capital, with labor constant, would normally be accompanied by a change of the total product in the same direction of 0.29 per cent. The standard error of the parameter k, is $\pm$ .065.

VI. A Comparison of the Computed and the Actual Indexes of Production.

A stringent test of the assumed relationship between the factors of production as expressed in the equation of production is to calculate an index of production from the equation and to compare it with the observed or actual index of production. Chart 2 presents both the actual and the computed indexes of production on a semi-logarithmic scale. Table 3 gives the values of the two indexes, and the deviations of the computed index (P') from the actual index

3. Douglas, The Theory of Wages, pp. 132–133.
4. For details of the method employed, see Section I, of the mathematical appendix in the succeeding installment.
5. If indexes P, L, and C are so chosen that their means are all equal to unity, then b will be equal to 1.0.

CHART 2.— ACTUAL AND COMPUTED INDEXES OF PRODUCTION (P AND P'), MANUFACTURING INDUSTRIES, VICTORIA

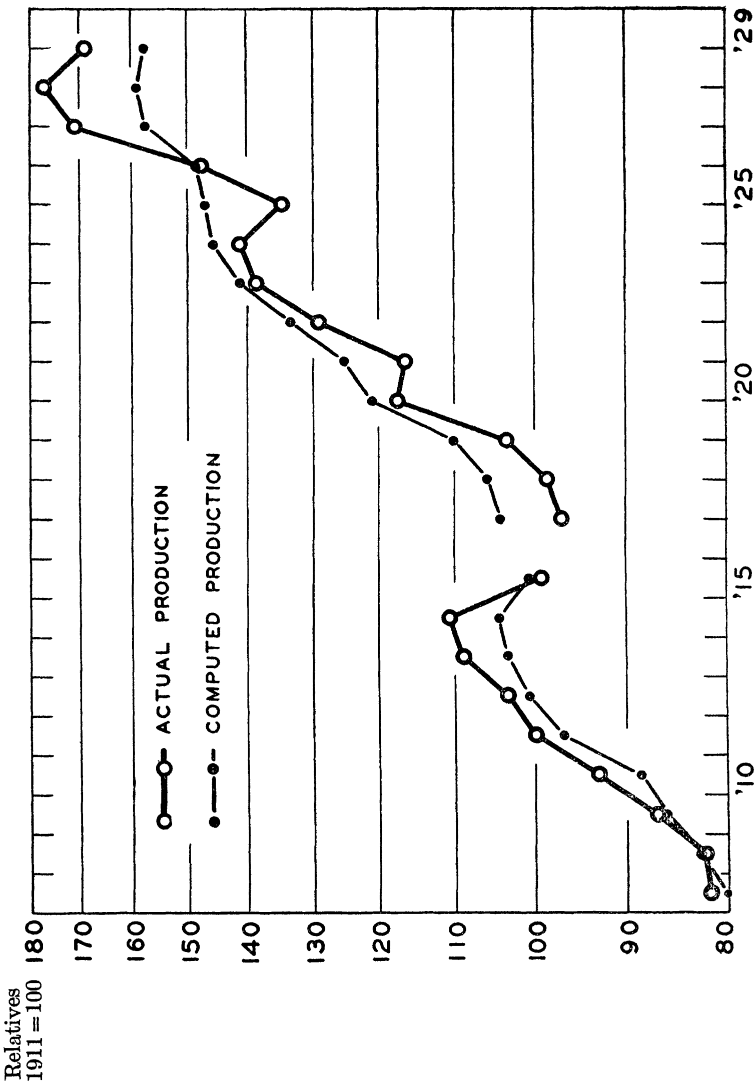


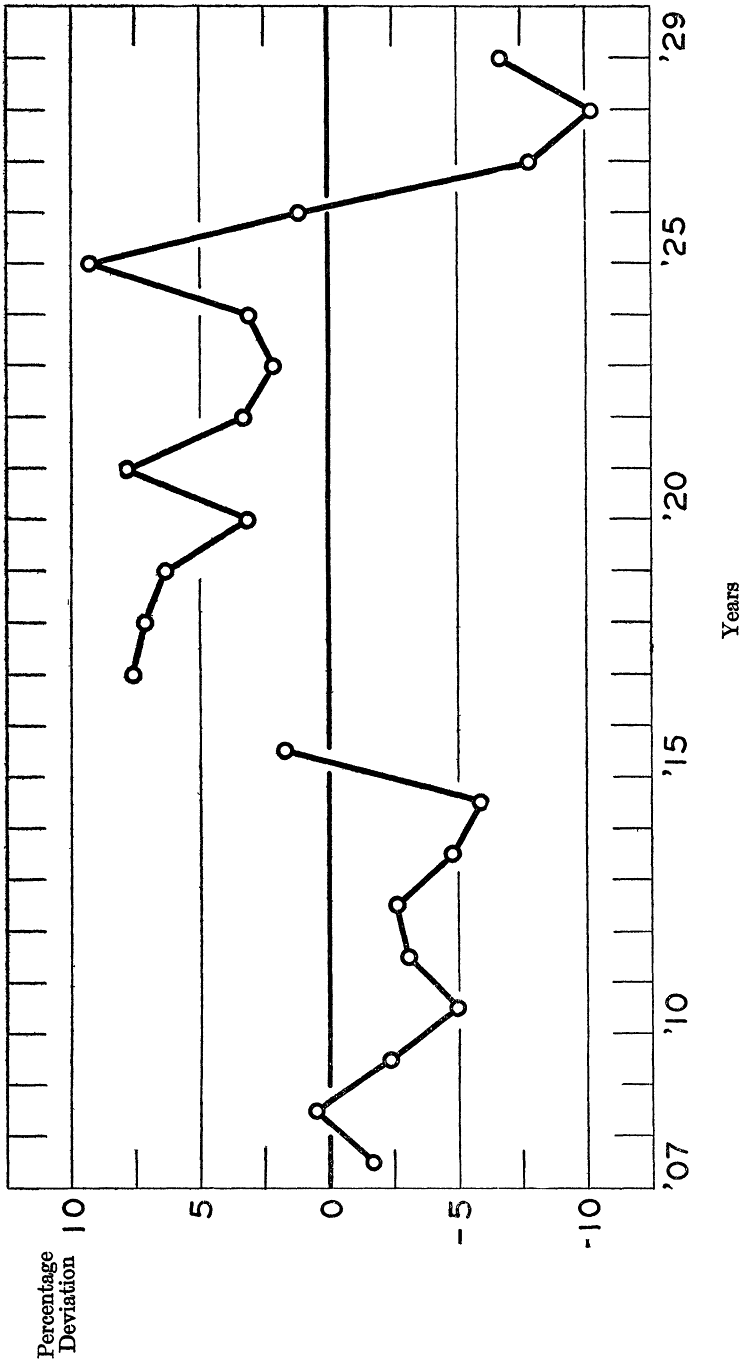
TABLE 3. — ACTUAL AND COMPUTED[1] INDEXES OF PRODUCTION IN MANUFACTURING INDUSTRIES, VICTORIA, AUSTRALIA, JANUARY, 1907–JUNE, 1929

Year	Actual Product P (1911 = 100) (1)	Computed Product P' (1911 = 100) (2)	$d = P' - P$ (3)	$d' = \frac{P'-P}{P}$ (4)
1907	81.6	80.2	− 1.4	− 1.7
1908	82.2	82.7	.5	.6
1909	87.0	85.0	− 2.0	− 2.3
1910	93.1	88.4	− 4.7	− 5.0
1911	100.0	97.0	− 3.0	− 3.0
1912	103.5	100.8	− 2.7	− 2.6
1913	108.5	103.4	− 5.1	− 4.7
1914	110.9	104.3	− 6.6	− 5.9
1915	99.1	100.8	1.7	1.8
1916–17	96.9	104.2	7.3	7.5
1917–18	98.8	105.9	7.1	7.1
1918–19	103.4	110.0	6.6	6.3
1919–20	117.5	121.1	3.6	3.1
1920–21	116.4	125.5	9.1	7.8
1921–22	128.7	133.1	4.4	3.4
1922–23	138.1	141.1	3.0	2.2
1923–24	141.2	145.5	4.3	3.0
1924–25	134.2	146.7	12.5	9.3
1925–26	147.3	148.8	1.5	1.0
1926–27	170.6	157.3	−13.3	− 7.8
1927–28	177.0	158.9	−18.1	−10.2
1928–29	168.6	157.3	−11.3	− 6.7

Total deviations[2] without regard to sign = 103.15 percentage points.
Average deviation = 103.15/22 = 4.69 percentage points.
Total deviation with regard to sign = 53.19 − 49.96 = 3.23 per cent.
Average deviation = 3.23/22 = 0.15 per cent.

[1] Equation of production: $P = .9699\, L^{.7140}\, C^{.2860}$.
[2] Deviations computed from indexes carried to two decimal places.

Chart 3.—Percentage Deviations of the Computed from the Actual Production Index $\frac{(P'-P)}{P}$
Manufacturing Industries, Victoria



(P), in both arithmetic and percentage terms. The percentage deviations of P' from P are also shown in Chart 3.

It will be seen that the computed curve of production fairly closely approximates the actual curve, and that with the exception of certain years the deviation is not great. In only two years was the difference between the two more than 9 per cent, and in only eight years more than 6 per cent. For the period as a whole, the average deviation of the computed from the actual index amounted to 5.9 points and 4.7 per cent. This should be compared with an average deviation of 7.5 points and 4.3 per cent for the two comparable series in the United States. The coefficient of correlation between the computed and the observed series is $+.97$ for Victoria, which is exactly the same as that obtained for the United States.

Let us now analyze in greater detail the divergences of P' (the computed index) from P. In Chart 3 these are shown by years in percentage terms. In the analysis of the differences between these two series for the United States, we found that they were largely explainable by cyclical factors. Our index of capital, for example, measures the relative capital available, rather than that which was actually used. As a result, it does not reflect the idle capital during a depression nor the intensive utilization of capital during prosperity. While our index of labor (i.e. employment) does reflect cyclical fluctuations much more closely, it does not measure the added decrease caused in depressions by short-time or the increase in prosperity caused by over-time. It follows, therefore, that our index of capital and, to a lesser degree, our index of labor are relatively somewhat too high during the depression years and too low during the years of prosperity. We should consequently expect the computed index to be greater than actual production during the depression phase of the cycle, and to be less during the prosperity phase. This, as a matter of fact, is what we found, with certain exceptions, for the United States, and the differences between P' and P, therefore, tended to confirm rather than disprove the formula which we adopted.

The issue is somewhat more complicated in the case of Victoria. As will be seen from Chart 3, the period as a whole can be divided into three sub-periods: (1) the years from 1907 through 1914, when the computed index (except for 1908) was below the actual index; (2) the period from 1915 through June, 1926, when the computed index was above the actual index; and (3) the period from July, 1926 through June, 1929, when the computed index again fell below the actual index. These differences are also in large part (altho by no means wholly) explainable by cyclical factors. Thus, the years from 1907 through 1912 were, according to Thorp's Business Annals,[6] primarily prosperous ones for Australia, except for a recession in 1908. One would expect, therefore, that our index would be lower than the actual index during all of these years except 1908, when it would be higher. This is precisely what we find. The year 1913, however, was characterized by a recession, and our computed index would, therefore, be expected to be greater than the actual. It was, however, 4.7 per cent less, and this discrepancy cannot be explained in the customary manner. Since 1914 was characterized by a business revival, even tho this was short-lived, the fact that our computed index was below the actual one in that year is, on the whole, a confirmation of our method.

When we come to the war and immediate post-war years, we find what superficially seems to be the contrary of what was to be expected. This period, if judged by prices and profits, was one of apparent prosperity, and we might, therefore, expect the computed index to fall below P from 1915 to 1919 inclusive. The fact that it was higher might, therefore, seem to refute our explanation. In fact, however, this apparent contradiction was caused by the World War which began in 1914. Australia sent large numbers of men to the front, who were withdrawn from industry and had to be replaced by inferior and less-skilled substitutes. It was also necessary to develop new lines of manufacturing for which

6. W. L. Thorp and W. C. Mitchell, Business Annals, National Bureau of Economic Research (New York, 1926), pp. 319–329.

technical experience on the part of both management and men was largely lacking. These forces would in themselves lower the actual index from what we should normally expect, and might well be sufficient to give the computed index a higher position.[7] In addition, the difficulties of importing raw materials during the shipping shortage also lowered the actual production, particularly in the metals group. The slowness of demobilization and the incidental difficulties of reabsorbing the soldiers into industry also operated in 1919 to dampen down the actual index. Then came the depression of late 1920 and 1921, which helped to make P' higher than P for the years 1920–21 and 1921–22. As we should expect, the divergence was greater in 1920–21 than in 1921–22.

During 1922–23, a period of mild prosperity, the actual index approached, but did not reach, the level of the computed index. In 1923–24 the actual index was further below the computed index than in the preceding years, reflecting the downswing of business activity. A revival occurred in the first half of 1925, but this was not reflected in the production index for the fiscal year 1924–25, which fell to a level of 9.3 per cent below the computed index. Each of the seven represented groups of industries showed a decline during that fiscal year. In 1925–26 the actual production index rose 13 points above the preceding year, reflecting prosperity in 1925–26, but was still 1.0 per cent below the computed index for that year. This is what one would expect from the nature of the series, altho one might also have expected the computed index for 1925–26 to be below the actual index instead of 1.0 per cent above it. Continued prosperity which lasted until the latter part of 1929 is reflected in the rise of the actual production index above the computed index during the last three years of the period studied.

On the whole, therefore, the differences between the two series can be fairly well explained by the failure of the labor and capital series to measure the full effects of cyclical

7. This was what we found in the United States for 1918 and 1919

fluctuations, together with the war and post-war dilution of labor. It should be frankly confessed, however, that the differences for 1913, 1922–23, and 1924–25, and perhaps for 1925–26, are not accounted for by these factors.

To the degree that the deviations of the computed from the actual index were caused by cyclical factors, we may remove most of these disturbing influences by taking the trends of the two series and comparing them. To test this relationship, four-year moving averages were computed.[8] Because there were variations in the length of the cycle,[9] these moving averages do not fully remove all cyclical influences, and still include the war and post-war influences. The average deviations between the series are reduced by this use of the trends, being only 4.2 instead of 5.9 points, and 3.6 instead of 4.7 per cent. The average percentage deviation between similar trends for the United States over a 22-year period was 2.6 per cent, and for Massachusetts over a 35-year period 5.8 per cent.

The coefficient of correlation between the trends for Victoria was $+.979 \pm .010$, which was slightly higher than that for the two unadjusted series ($+.966 \pm .014$).[1] It is still true, however, that the differences between the adjusted series fall into the same three series of years as do the unadjusted series, indicating that there were basic and underlying swings in addition to the four-year cycle.

Some critics have argued that the relatively close agreement which we discovered to exist in the United States between P and the index P' as computed from the formula

$$P' = 1.01\, L^{75} C^{25}$$

8. The four-year moving averages of the actual and computed indexes of production, and the deviations of the trend of one from the trend of the other, are shown in Table V of the mimeographed supplement.

9. According to Thorp and Mitchell, op. cit., p 51, the lengths of the cycles from 1908 to 1924 were 5, 2, 6, and 3 years respectively.

1. The standard errors of the correlation coefficients shown in the article were calculated by the short formula, and are subject to the limitations pointed out by R. A. Fisher, Statistical Methods for Research Workers (3d Edition, London, 1930), pp. 158–161, and p. 176.

was simply due to the fact that the trends of L, C, and P all moved upward together. Consequently the correlation between P and P' is declared to be nonsensical. As one of the present authors has stated elsewhere, however, if there is no functional relationship between labor and capital, on the one hand, and product, on the other, then economic science has no meaning; and to insist that in any study of causal relationships the respective trends must first be eliminated is often to throw the baby out with the bath. For not only are the relative slopes of the trends very important, but, in the case mentioned, product is certainly not independent of the quantities of the labor and capital factors. On the contrary, product is affected by the relative quantities of labor and capital. It is important, therefore, to determine, as we have done, the inter-relationship between the trends of L and C and that of P.

In order to round out the analysis, however, it is desirable to compare the deviations of both P' and P from their respective trends, to see if their cyclical fluctuations correspond. This has been done, and the results are shown in Table 4 and Chart 4. Two points stand out. (1) The amplitude of fluctuations of P (the index of actual production) about its trend are greater than the fluctuations of P' (the computed index) from its trend. The standard deviation of the former was 4.6, and of the latter 1.9. This difference was caused by the fact that the capital index was not sensitive to cyclical influences, while the labor index was not perfectly sensitive. This naturally made the computed index less sensitive than the actual. (2) The coefficient of correlation between the deviations of these indexes from their respective trends was $+.784 \pm .091$. Since the probable error was only about one-ninth of the coefficient, this meant a relatively high degree of correspondence between these two indexes, even in the case of short-run and cyclical fluctuations.[2] There were, in fact, only four years in which

2. In the United States, the coefficient of correlation between similar series was $+.94 \pm .02$. In New South Wales, it was $+.51 \pm .17$. The computed Production Index in New South Wales was very close to the

TABLE 4. — DEVIATIONS OF ACTUAL AND COMPUTED PRODUCTION INDEXES FROM THEIR RESPECTIVE FOUR-YEAR MOVING AVERAGES, VICTORIA, AUSTRALIA, JANUARY, 1909–JUNE, 1927

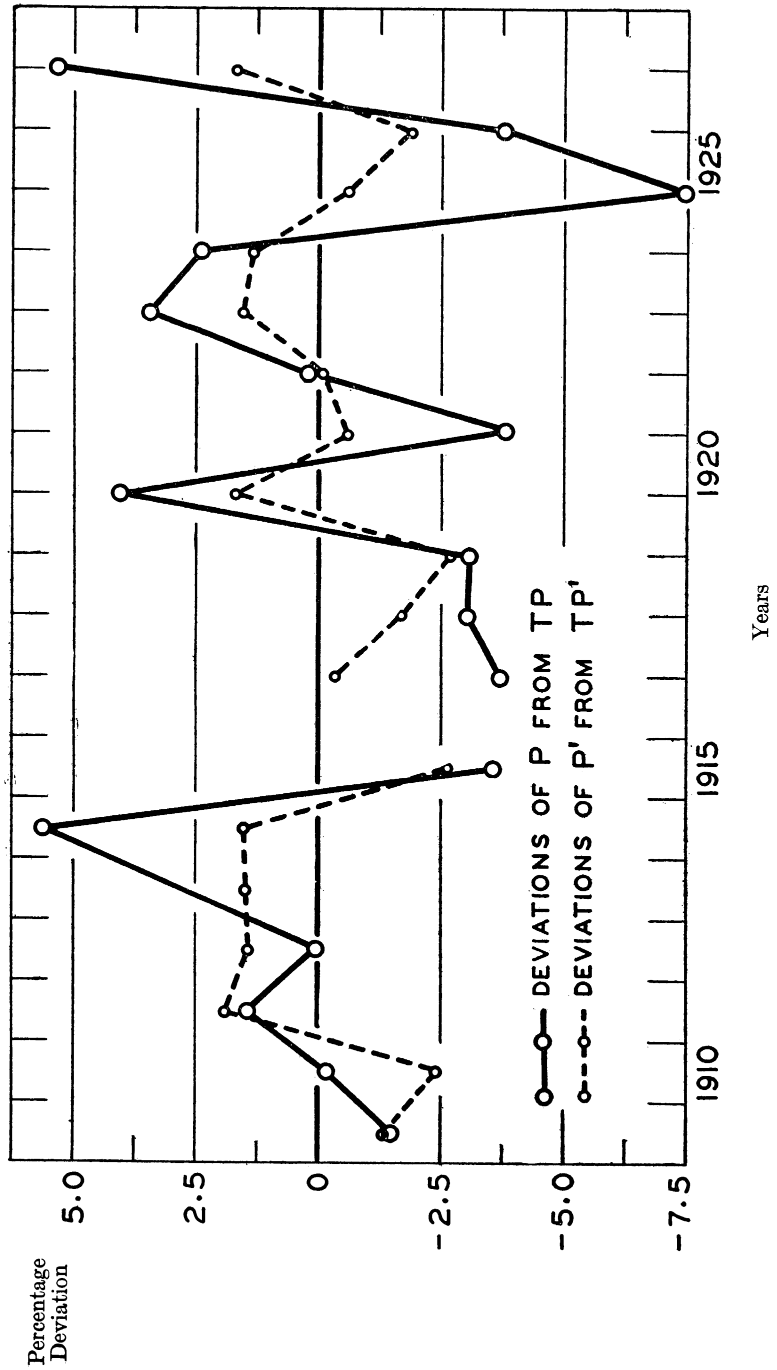
Year	P–trend P (1)	P'–trend P' (2)	Percentage deviations $\frac{P\text{–trend } P}{P}$ (3)	Percentage deviations $\frac{P'\text{–trend } P'}{P'}$ (4)
1909	− 1.3	− 1.2	− 1.5	− 1.4
1910	− .2	− 2.1	− .2	− 2.4
1911	1.4	1.9	1.4	2.0
1912	(1)	1.4	(2)	1.4
1913	2.9	1.5	2.6	1.5
1914	6.2	1.6	5.6	1.5
1915	− 3.5	− 2.6	− 3.6	− 2.6
1916–17	− 3.6	− .3	− 3.7	− .3
1917–18	− 3.0	− 1.8	− 3.0	− 1.8
1918–19	− 3.2	− 3.0	− 3.1	− 2.7
1919–20	4.7	2.1	4.0	1.7
1920–21	− 4.4	− .8	− 3.8	− .6
1921–22	.6	− .2	.5	− .1
1922–23	4.7	2.1	3.4	1.5
1923–24	3.3	1.9	2.4	1.3
1924–25	−10.1	− .9	− 7.5	− .6
1925–26	− 5.5	− 2.5	− 3.8	− 1.6
1926–27	9.1	3.1	5.3	2.0

		(3)	(4)
Total deviations without regard to sign[3]		55.43	27.06
Average deviation without regard to sign		3.08	1.50
Deviations with regard to sign:			
	positive	25.29	12.85
	negative	−30.14	−14.21
	sum	− 4.85	− 1.36
	average	− 0.27	− 0.08

[1] Less than one-tenth of one point.
[2] Less than one-tenth cf one per cent.
[3] Deviations computed from indexes carried to two decimal places.

CHART 4.— PERCENTAGE DEVIATIONS OF P AND P' FROM THEIR TREND LINES, MANUFACTURING INDUSTRIES, VICTORIA

(Trend = four-year moving average)



the movement of the indexes from their respective trends was in the opposite direction. These years were 1910, 1916–17, 1917–18, and 1925–26.

The Victorian data did not, however, corroborate the experience of the United States, where it was found that the actual Production Index (P) lay closer to the computed index (P') than it did to its own moving average, the standard deviation of the former ($P'-P$) being 8.7 and for the latter ($P-$trend of P), 11.7. In Victoria, on the other hand, the standard deviation of the difference between the two series was 7.4, while, as stated above, it was only 4.6 between P and its trend values.

In a concluding article we shall (1) show the marginal productivity curves which we have derived for Victorian manufacturing; (2) compare the actual distribution of the value product and the movement of real wages with what we would expect from the formula in question; and (3) discuss the theoretical issues which are raised both by the results and by this method of approach.

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actual Production Index Both showed trend with little cyclical movement. Deviations from trend were, therefore, less significant in this case than in the other studies.

[To be concluded]